A simple theory of landslide failure and size distribution

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Abstract.
Landslides triggered in large numbers by heavy rainfall, earthquake strong ground motion or snowpack ablation have number-area distributions with remarkably similar negative power-law tails. There may be a simple physical explanation for this behaviour. Each landslide begins with slip at a weak point; as the slip surface propagates, fluctuations occur in the forces driving and resisting the propagation. At some point the sum of these fluctuations reaches zero, friction takes the upper hand, and the slip surface stops spreading. The probability that a slip surface triggered at a random point will grow to reach a particular size is a solution of the stochastic differential equation that describes the series of fluctuations as a mixed Gaussian diffusion process. This solution is robustly consistent with landslide size distributions observed in nature. Its steep power-law tail is the result of fluctuations in the net driving force that scale with the spreading slip area, while the average size landslide is determined by fluctuations at the growing perimeter. The origin of the fluctuations is perhaps the variability of topographic relief with scale.

Introduction
Considerable attention has been given to the power-law scaling of number-size distributions in nature [Mandelbrot, 1982] and data gathered on large populations of landslides [Guzzetti et al., 2002; Hovius et al., 1997, 2000; Harp and Jibson, 1996; Stark and Hovius, 2001] have provided an excellent example of this phenomenon. No adequate explanation for landslide size scaling has been forthcoming, beyond the idea that power-law scaling could be diagnostic of landscapes in a self-organized critical (SOC) state [Guzzetti et al., 2002; Bak, 1990]. There are several problems with this idea. The most severe is that while SOC models yield power-law distributions in space and time, the presence of a power-law size distribution alone is by no means indicative of a SOC system - many other physical processes can bring about power-law scaling. Nor does a SOC explanation provide much insight into the physics of the slope failure process, beyond predicting temporal correlations in landslide failure for which there is no evidence. Furthermore, SOC models ignore the first order scaling effect of spatial interaction between hillslopes and channels, which must be addressed if a dynamic, globally self-organized critical state is to be inferred in place of a simpler, local mechanism.

Model
A more appealing explanation may lie in the growth process of individual landslip surfaces. Failure of a hillslope occurs when the weight of soil or bedrock exceeds effective friction and cohesion at a weak point, typically a few metres below the hillslope surface, and generally because a rise in pore fluid pressure or seismic strong ground motion reduces the normal stress to a critical level. The landslide grows as the slip surface propagates (upslope, downslope and laterally), and continues to do so until the forces driving propagation are overwhelmed by friction. Surface breaking and runout will not be considered here.

The growth process and arrest can be modelled as a sequence of fluctuations in the imbalance of driving and resisting forces $y(x)$ from an initial level $y(0) = \varphi_0$. At least two kinds of fluctuation need to be considered in order to explain the number-size scaling observed in nature: first, variations in net driving force $dy_0$ that arise at the landslide perimeter as the slip spreads in area by $dx$; second, variations in net driving force $dy_1$ that occur across the whole deforming landslide body and scale with the slip area $x$. A simple model treats these fluctuations as uncorrelated, Gaussian white noises, $dy_0 = \sigma_0 \, dW_0$ and $dy_1 = \sigma_1 \, x \, dW_1$, and their sum gives the stochastic differential equation [Gardiner, 1985] for changes in the net driving force $y(x)$,

$$dy = \sigma_0 \, dW_0 + \sigma_1 \, x \, dW_1 .$$

The constants $\sigma_0$ and $\sigma_1$ are the amplitudes of the respective fluctuations (with units equivalent to $y/\sqrt{x}$ and $y/\sqrt{x^3}$ respectively) while $W_0(x)$ and $W_1(x)$ are the corresponding Wiener processes (Brownian motions).

Fluctuations in the net forces driving the spread of a landslide in nature will arise for number of reasons, such as variations in soil and rock shear strength and cohesion, in pore fluid pressure, in root mass strength, in the geometry of the slip surface, through deformation within the landslide body, and so on. Perhaps the strongest fluctuations of all will arise as a result of the variations
in topographic relief that a slip surface will encounter as it spreads. In any case, the utility of this growth model is its simplicity, and for simplicity it is useful to treat the fluctuations in this mathematically abstract fashion so that a quantitative treatment of the problem can proceed.

As an illustration of the stochasticity of the model growth process, the slip histories of three different model landslides are shown in figure (1). In each case, slip begins at approximately the same level of net driving force \( \varphi_0 \), but as each landslide grows, the differing series of fluctuations drive the trajectories of net driving force into radically different patterns. As a result, there is broad variation in the maximum size \( x = A \) that a landslide may reach. The model size distribution for landslides is the probability density \( p(A) \) that slip will grow to reach an area \( A \). This is the solution of equation (1) for first arrivals at \( y = 0 \).

**Analytical solution**

An analytical solution of the stochastic differential equation (1) can be found by transforming it into a Fokker-Planck equation [Gardiner, 1985]. This transformed version represents the flow of probability \( p(x, y) \) as a diffusion equation with non-constant diffusivity,

\[
\partial_x p(x, y) = \frac{1}{2} \left( \sigma_0^2 + \sigma_1^2 x^2 \right) \partial_y^2 p(x, y).
\]  

(2)

The first arrival probability \( P(A = \inf \{x|y = 0\}) \), which is the likelihood that a landslide will freeze at a given area \( x = A \), can be obtained in a number of ways, such as the method of images [Seshadri, 1993],

\[
p(A) = \frac{s}{\sqrt{2\pi}} \left( \frac{1 + A^2/ \mu^2}{(1 + A^2/3 \sigma^2)^{3/2}} \right) A^{-3/2} \times \exp \left( -\frac{s^2}{2A(1 + A^2/3 \sigma^2)} \right) \]  

(3)

where \( r = \sigma_0/\sigma_1 \) and \( s = \varphi_0/\sigma_0 \). Figure (2) illustrates the two components of this analytical solution (for \( \sigma_0 \) \( dW_0 \) only and for \( \sigma_1 x \) \( dW_1 \) only) together with the solution for the full mixed diffusion (heavy blue curve). The simple \( \sigma_0 \) diffusion alone gives a Lévy \( \alpha = \frac{1}{2} \) distribution with a power-law tail with exponent \( -\frac{3}{2} \), while the growing fluctuation \( \sigma_1 \) diffusion gives the steeper \( -\frac{5}{2} \) tail that dominates the full model solution. The \( \sigma_0 \) diffusion component dominates decay towards the origin.

It appears that every landslide data set (figure (3)) with good temporal control shows this behaviour [Guzzetti et al., 2002; Stark and Hovius, 2001], irrespective of the nature of the triggering mechanism and for a variety of landscapes. Second, the power-law tail will retain the same steepness of \( \alpha = \frac{1}{2} \) regardless of the level of initial driving force and irrespective of the amplitude of the fluctuations, unless \( \sigma_1 = 0 \), when the size distribution reduces to a simple Lévy \( \alpha = \frac{1}{2} \) density [Seshadri, 1993] (figure (2)). Third, the model distribution is very parsimonious, with only two free parameters, \( r = \sigma_0/\sigma_1 \) and \( s = \varphi_0/\sigma_0 \). Fourth, equation (3) suggests that a characteristic length scale \( L_c \), located roughly at the mode of the distribution \( p(A) \), can be defined as \( L_c \equiv s/\sqrt{2} \).

**Numerical solution**

The model can be solved numerically in a straightforward fashion by computing a large number of trajectories of \( y(x) \) using a good pseudo-random number generator [James, 1990]. In the examples to be presented here (figure (3)), an ensemble of \( 10^5 \) in-
Results

The shape of the model size distribution matches observed landslide size distributions very well, as figure (3) demonstrates. Landslide data from four regions are shown in these graphs, illustrating in each case a different triggering mechanism for slope failure: (a) 2226 shallow and 1628 deep landslides triggered in Umbria, central Italy, following the melting of a heavy snowpack in 1997 [Guzzetti et al., 2002]; (b) 11111 landslides triggered by the 1994 Northridge earthquake (M=6.7) [Harp and Jibson, 1996]; (c) 3986 landslides from the Whataroa catchment, located in the western Southern Alps of New Zealand, triggered over a period of about 50 years by a mixture of convective and cyclonic storms [Stark and Hovius, 2001]; (d) 1013 landslides triggered over several decades during heavy monsoon rains, primarily during typhoons, in a high relief part of the Hualien watershed in the Central Range of Taiwan [Hovius et al., 2000]. Estimated probability densities for the observed data (open circles) are compared with model landslide size distributions (numerical solutions of the stochastic differential equation (1): solid red circles; analytical solutions of the Fokker-Planck equation (2): heavy blue curve). Model parameters \( r = \sigma_0/\sigma_1 \) and \( s = \varphi_0/\sigma_0 \) were in each case adjusted to give results consistent with the data, while the power-law shape of the tails of the distributions is fixed at \( \alpha = 3/2 \) by the phenomenology of the differential equation (1) alone.

Best-fit model parameters for these data sets are summarized in table (1), where they are expressed as length scales. Some care must be taken in interpreting these as real length scales, because the censoring (undersampling) effect involved in the mapping of

\[
L_c^2 = \frac{5}{2} \left( \frac{\varphi_0}{\sigma_0} \right) \left( \frac{\sigma_1}{\sigma_0} \right)
\]

Data
Model
Simulation

![Figure 3. Observed (open circles) and model landslide size distributions (numerical solutions - solid red circles; analytical solutions - heavy blue curve), expressed as probability densities. (a) snowpack triggered landslides (2226 shallow and 1628 deep), Umbria, central Italy; (b) 11111 landslides triggered by the 1994 Northridge earthquake, Northridge, CA; (c) 3986 landslides from Whataroa catchment, western Southern Alps, NZ; (d) 1013 landslides triggered largely by typhoons, Hualien, Taiwan.](image-url)
The growth model is to describe the ultimate size of an arbitrary physical meaning of the model fluctuations, and the key to this is the driving force experienced by a growing model landslide are an abstraction of the ensemble of variations in mechanical and topographic properties that every possible landslide will undergo. The series of fluctuations in net graphic properties that every specific point of initial failure. Topographic barriers such as ridges and valleys across a range of scales) to landslide growth. Nevertheless, the analyses of the Umbria and Northridge data (figure (3)) demonstrate that total landslide area can be used in place of source area with the important caveat that any inferred length scales will be slightly enlarged relative to their true values.

## Discussion

The physical significance of the model parameters rests on the physical meaning of the model fluctuations, and the key to this is the mean field nature of the model. Remember that the task of the growth model is to describe the ultimate size of an arbitrary centered, initial slope failure. The series of fluctuations in net driving force experienced by a growing model landslide are an abstraction of the ensemble of variations in mechanical and topographic properties that every possible landslide will undergo. The specific variations in substrate strength and landscape morphology surrounding each specific point of initial failure are subsumed into a "mean field" of probable fluctuations in net driving force for a generic point of initial failure. Topographic barriers such as ridges and valleys are treated as equivalent to mechanical barriers such as jumps in soil cohesion or drops in pore fluid pressure.

In this interpretation, the growing fluctuations \(\sigma_1 dW_1\) likely reflect the generalized, mean-field effect of topographic barriers (ridges and valleys across a range of scales) to landslide growth. An alternative explanation is that they reflect variations in mechanical properties of the deforming landslide that scale with the growing slip area. Further study is required to iron out this ambiguity. If the topographic explanation is correct, then the characteristic scale \(L_c\) is probably closely related to the hillslope length scale, modulated by the type of landslide failure (witness the different values of \(L_c\) for the shallow versus deep events in the Umbria data). The constant amplitude fluctuations \(\sigma_0 dW_0\) likely reflect hillslope-scale variations in both mechanical properties and surface relief.

## Conclusions

In summary, a simple stochastic theory for spreading slope failure is proposed as an explanation for the shape of landslide size distributions. The quantitative form of this theory is a mixed diffusion equation that describes the fluctuations in the driving and resisting forces acting on a spreading landslide. The model landslide size distribution is the solution of this diffusion equation for when frictional forces overcome driving forces. The theory predicts a unimodal distribution bounded about the modal landslide area by an asymptotically power-law decay towards larger landslide areas and an exponential decay towards smaller areas. The model power-law decay has a constant steepness with an exponent of \(\frac{3}{2}\) irrespective of the triggering mechanism of the landslides. This prediction is borne out by observed distributions of landslides triggered by a variety of mechanisms ranging from snowpack ablation, to heavy rainfall, to earthquake strong ground motion.

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## References


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