Analysis of Erosion Thresholds, Channel Networks, and Landscape Morphology Using a Digital Terrain Model

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ABSTRACT

To investigate the linkage between erosion process and channel network extent, we develop two simple erosion threshold theories driven by a steady-state runoff model that are used in the digital terrain model TOPOG to predict the pattern of channelization. TOPOG divides the land surface into elements defined by topographic contours and flow lines, which can be classified as divergent, convergent, and planar elements. The calibration parameter for the runoff model is determined using empirical evidence that the divergent elements which comprise the ridges in our study area do not experience saturation overland flow, whereas the convergent elements in the valleys do during significant runoff events. A threshold theory for shallow landsliding predicts a pattern of instability consistent with the distribution of landslide scars in our 1.2 km² study site and confirms the interpretation, based on field observations, that indicate the steeper channel heads to be at least partially controlled by slope instability. Most sites of predicted and observed slope instability do not, however, support a channel head, hence landslide instability alone is not sufficient for channelization. In contrast, most elements predicted to be eroded by saturation overland flow coincide with the observed location of the channel network. In addition, areas of predicted downslope decrease in relative sediment-transport capacity were found to correspond to locations where channels became discontinuous. The topographic threshold given by the saturation overland flow erosion theory varies with the third power of critical boundary shear stress, suggesting that critical shear stress, although difficult to quantify with much precision in the field, is a dominant control on the extent of the channel network where saturation overland flow is significant. Current extent of the channel network in our field site, for example, may best be explained as resulting from grazing-induced reduction in surface resistance.

Introduction

Landscape dissection is controlled by the tendency for runoff to channelize into avenues of concentrated erosion (e.g., Horton 1945; Smith and Bretherton 1972). Field studies have revealed that the upslope extent of the channel network may be well defined by an inverse relationship between drainage area and slope (Montgomery and Dietrich 1988, 1989, 1992). These studies have also shown that at the channel head, there is typically a process change, upslope of which mass wasting and diffusive processes predominate and downslope of which runoff-driven incision occurs (e.g., Dietrich and Dunne 1993). Hence, in a given catchment, there appears to be a threshold of erosion resistance which sets the location of a channel head at a specific drainage area and local slope, and therefore determines the extent of the channel network in a watershed. This concept has been explored in a landscape evolution model by Willgoose et al. (1991).

Acquisition of high-resolution digital elevation data allows a comparison between the location of channel networks mapped in the field and networks predicted by process-based theories. Other than the comparison between predicted and observed ephemeral gullies in an agricultural field by Moore et al. (1988a), however, we know of no such studies. Numerical models can convert the elevation data into a digital surface upon which water and sediment can be routed. At present, however, digital elevation data, even at very high resolution (say 1 m grid spacing) are too sparse to capture the local topography around typical small channel...
heads, which often are only decimeters in size at their tips. Process-based models for channel networks, then, are best thought of as attempting to define where in the landscape there would be a tendency for erosion to channelize, rather than as attempting to predict the size and downstream topographic evolution of discrete channels that collect to form the network.

In this paper we build upon a recent study (Dietrich et al. 1992) to propose two simple threshold of erosion models that, based on field observations, appear appropriate for explaining channel initiation in a small watershed where the full extent of the channel network has been mapped. We develop these theories in a form which can be easily tested using digital elevation data and a simple hydrologic routing model. Our analysis suggests that both the upslope extent and discontinuous nature of the channel network in our field area in Marin County, California, can be explained primarily by saturation overland flow erosion. Shallow landsliding, while active at many of the steeper channel heads, also occurs in areas that do not have sufficient runoff to maintain a downslope channel. Hence landsliding does not uniquely define channel-head locations. The mechanistic analysis proposed here, although crude, does strongly suggest how the channel network in our study area has responded to climatic change and landuse. It also leads to further testable hypotheses regarding erosion thresholds.

Theory

Three simple theories for predicting runoff, slope instability, and erosion by saturation overland flow. Although these theories were reported by Dietrich et al. (1992), their derivations were not given. These theories are developed here in a form to take advantage of two topographic attributes easily determined from digital elevation data: drainage area per unit contour length, and local ground slope.

Runoff Model. In well-vegetated semi-arid to humid landscapes where precipitation intensity does not exceed the soil infiltration capacity, runoff occurs by subsurface and saturation overland flow (e.g., Dunne 1978). Erosion thresholds will be crossed during precipitation events that elevate pore pressures associated with subsurface flow to the point of mass failure of the ground or where subsurface flow returns to the surface and combines with incoming precipitation to generate saturation overland flow of sufficient depth to cause surface incision of the ground. On exposed cuts such as in channel heads, seepage erosion may require a critical hydraulic gradient to displace surface materials (Dunne 1990, Montgomery 1991), but this process will not be considered here.

Although erosion events are clearly driven by storms of varying intensity and duration that occur on watersheds of seasonally varying antecedent conditions, the modeling of such unsteady rainfall-runoff processes introduces considerable complexity and several calibration parameters for which spatial and temporal variability is difficult to define. Given that we can only ask whether a particular place in a landscape is likely to experience hydrologic conditions favorable for a threshold of erosion to be exceeded, rather than make specific predictions about the size and dynamics of individual channels, it seems appropriate to keep the hydrologic driving function as simple as possible, while still retaining the essential physics. Here we use a steady-state runoff model that can be interpreted as mimicking the topographic dependence of runoff during transient storm events. This makes evaluation of the predicted storm runoff difficult, but we suggest that the landscape topography itself places some constraint on the magnitude of the saturated flow response. We will also assume that transmissivity, saturated conductivity, and the ground resistance to runoff and erosion are spatially constant.

Following the formulation proposed by O'Loughlin (1986) for subsurface flow, we can write the conservation of mass for steady-state, shallow subsurface flow and saturation overland flow as follows:

\[ qa - TMb = udb. \tag{1} \]

As shown in figure 1, \( a \) is the catchment draining across a contour of length, \( b \). The total runoff per unit area, \( q \), is equal to the precipitation, \( p \), minus the evaporation, \( e \), minus deep drainage, \( r \), i.e., \( q = p - e - r \). As O'Loughlin (1986) correctly points out, even in the steady-state case, \( p, e, \) and \( r \) need not be spatially constant, although we assume so here. The shallow subsurface flow modeled here is assumed to flow parallel to the ground surface and at saturation is equal to the transmissivity, \( T \), times the surface slope, \( M \), times the contour length, \( b \). Here \( M \) equals \( \sin H \). Runoff that cannot be transmitted in the subsurface must travel overland, and does so with a mean velocity of \( u \) for a depth, \( d \), across the contour length, \( b \). Hence, equation (1) simply states that the difference between the total runoff and the saturated
subsurface flow is the overland flow at steady-state conditions.

According to equation (1), the ground will saturate everywhere that these conditions hold:

$$\frac{a}{b} \geq \frac{T}{q} M.$$  \hspace{1cm} (2)

Equation (2) can be rearranged to state that where the topographic ratio, $a/(bM)$, is greater than or equal to the hydrologic ratio, $T/q$, the ground will saturate. In essence, $a/(bM)$ is the topographic term that expresses the tendency for flow convergence (high $a/b$) and for the flow to travel quickly as shallow subsurface flow (large $M$) rather than as saturation overland flow (small $M$). Large areas draining to gentle slopes (i.e., where this ratio is large) are prone to saturation. The hydrologic ratio, $T/q$, can be thought of as the ability of the subsurface to transmit flow relative to the applied runoff.

Hence, where this ratio is small, the ground tends to saturate.

If the hydrologic parameter, $T/q$, is roughly spatially constant for a given storm, then a survey of the upslope extent of saturation at points throughout a watershed should find these points having the same value of $a/(bM)$ (e.g., Kirkby 1978; Beven and Kirkby 1979). Equation (2), when used in combination with knowledge of the landscape form derived from a digital elevation model, will also prove useful in establishing the value of $T/q$, as discussed later.

**Slope Stability Threshold.** Shallow landsliding occurs when pore water pressure reduces the frictional strength of the soil to the point where the shear stress due to the downslope weight of the soil exceeds the strength. We assume that the zone of shallow surface storm runoff is confined to a cohesionless (at failure) soil overlying a less conductive, but stronger bedrock. Hence, in figure 1, $z$ defines not only the maximum thickness of shallow surface flow, but also the thickness of the shallow, potentially unstable surface layer. The ground does not have to be saturated for failure to occur; hence the local thickness of saturated flow, $h$, can be less than or equal to $z$. If $T = Kz \cos \theta$, in which $K$ is the saturated conductivity, then for subsurface flows equal to or less than saturated, the ratio of the actual runoff to maximum subsurface flow is:

$$\frac{qa}{bTM} = \frac{K \sin \theta \cos \theta}{K \sin \theta \cos \theta} = \frac{h}{z}.$$ \hspace{1cm} (3)

Because our hydrologic model assumes flow parallel to the ground surface, excessive pore pressure associated with exfiltrating gradients cannot be modeled. The thickness of the saturated zone required to convey the imposed runoff varies from a small value up to $z$, the thickness of the conducting layer; hence $h/z$ varies from nearly zero to 1.0.

For cohesionless material, the infinite slope stability model can be written as:

$$\rho_s z \tan \theta = (\rho_s h - \rho_w h) \tan \phi$$

and with substitution of equation (3) for $h/z$ this becomes:

$$\frac{a}{b} \geq \frac{\rho_s}{\rho_w} \left(1 - \frac{\tan \theta}{\tan \phi}\right) \frac{T}{q}.$$ \hspace{1cm} (4)

This states that instability will occur when the area per unit contour length is greater than or equal to the product of four terms: the bulk density ratio
of wet soil \( (\rho_s) \) to water \( (\rho_w) \), the ratio of the tangent of the ground surface to the angle of internal friction \( (\tan \phi) \), the hydrologic ratio \( (T/q) \), and the ground slope \( (\sin \theta) \). Although it would be more realistic to include a cohesion term in equation (4), this requires additional information about soil depth, or at least the depth of rooting. Again, for simplicity, here the contribution of cohesion is neglected. Because of the condition of flow parallel to the ground surface, slopes greater than or equal to the friction angle are chronically unstable and would be in areas where bedrock is exposed. Also, for slopes with \( \tan \theta \leq [(\rho_s z - \rho_w h)/(\rho_s z)] \cdot \tan \phi \), the ground is stable even if saturated.

**Threshold of Erosion by Saturation Overland Flow.** In general, we can write the conservation of mass equation for erosion due to sediment transport as:

\[
\nabla \cdot \vec{q} = -\rho_s \frac{\partial z}{\partial t}
\]

in which \( \vec{q} \) is the sediment transport vector, \( \rho_s \) is the dry bulk density of the soil surface, \( t \) is time, and \( z \) is the local elevation of the ground surface. Sediment transport by running water varies in proportion to boundary shear stress (e.g., Slattery and Bryan 1992, Dunne 1991), therefore divergence of the boundary shear stress (in excess of some critical value) will induce erosion. The threshold of erosion by overland flow, however, is not necessarily equivalent to the threshold of channel incision by that flow. This was clearly the case in rill incision on both an unvegetated surface (i.e., Slattery and Bryan 1992), and on poorly vegetated surfaces [Dunne and Aubrey 1986]. Dunne [1980] has argued that one reason for this stability, at least in coarser sediments, is the diffusive behavior of rain-splash transport in shallow sheet flow, which would tend to damp the tendency for the flow to incise. Based on flume experiments in somewhat cohesive materials, there is evidence that although the lateral instability of the flow develops, it does not cut a channel (or rill) until the flow has become approximately supercritical and causes the formation of an upslope propagating knickpoint (Slattery and Bryan 1992).

Because of the high resistance to surface erosion in the well-vegetated areas where saturation overland flow occurs and the resulting significant flow depth needed to generate boundary shear stress to cause sediment transport, we will assume that once a critical boundary shear stress \( (\tau_c) \) is exceeded, the flow will incise and form a channel. This is particularly true where the surface, due to roots, stems and decayed organic matter, is considerably more resistant than the immediate subsurface. As long as the actual relationship between boundary shear stress \( (\tau_h) \) and sediment transport rate is strongly non-linear, as might be expected in this case, the characterization of the surface as having a threshold boundary shear stress is reasonable. If we also assume that sediment transport, \( q_s \), is proportional to an excess boundary shear stress, as is usually shown for sand and coarser material (e.g., Yalin 1972), then

\[ q_s \alpha (\tau_h - \tau_c)^n \]

and channels would be expected to form when \( (\tau_h \geq \tau_c) \) and tend to thin and disappear if \( (|d(\tau_h - \tau_c)|/\partial x| < 0 \). On poor- to well-vegetated surfaces, the availability of sediment for transport is limited, so the actual transport rate will be well below the transport capacity estimated from an excess boundary shear stress equation. Nonetheless, we propose that the tendency for scour or deposition will still respond to spatial change in excess of shear stress.

**Boundary shear stress in nonaccelerating flows** can be written as:

\[ \tau_h = \rho_w g d M. \quad (5) \]

Gravitational acceleration, \( g \), and fluid density, \( \rho_w \), are known, and the local slope can be determined from a sufficiently high-resolution digital elevation model. The flow depth must be calculated based on runoff rate and surface roughness. Rewriting equation (1) to solve for flow depth,

\[ d = \frac{1}{ub} (qa - TMb), \]

multiplying each side by \( \rho_w g M \), and substituting into (5), gives

\[ \tau_h = \frac{\rho_w g M}{ub} (qa - TMb). \quad (6) \]

To eliminate the velocity term in (6), we use a form of the Darcy-Weisbach equation often employed in overland flow studies (e.g., Dunne and Dietrich 1980),

\[ u = \left( \frac{2gdM}{f} \right)^{0.5}. \quad (7) \]

For turbulent flow the friction factor, \( f \), is constant but for laminar flow the friction factor varies in-
versely with Reynolds number, i.e., $f = K v / u d$ in which $v$ is the kinematic viscosity and $K$ is a constant for a given roughness condition. Using the frictional relationship given in equation (7), the boundary shear stress for turbulent flow is

$$\tau_b = \rho_w |Mg|^{2/3} \left( \frac{f}{2} \right)^{1/3} \left( q \frac{a}{b} - TM \right)^{2/3},$$

whereas for laminar flow

$$\tau_b = \rho_w |Mg|^{2/3} \left( \frac{K v}{2} \right)^{1/3} \left( q \frac{a}{b} - TM \right)^{1/3}.$$  

Setting equations (8) and (9) equal to the critical shear stress and solving for the topographic condition where the critical shear stress is equaled or exceeded gives

$$\frac{a}{b} \geq \frac{1.41 \tau_c^{1/2} + T}{M}$$

for turbulent flow and

$$\frac{a}{b} = \frac{2 \tau_c}{\rho_w K v g M^{2/3} q + T}$$

for laminar flow.

This analysis shows that if channel initiation by saturation overland flow can be characterized as occurring once a critical boundary shear stress is exceeded, then the area per unit contour length upslope of the channel varies with critical shear stress and inversely with roughness, precipitation and, if critical shear stress is sufficiently large, inversely with ground slope. If the critical shear stress is very small, then equations (10) and (11) reduce to equation (2), that is, channel initiation is equivalent to ground saturation. Montgomery (1991) has suggested that in some instances ground saturation may be the appropriate criteria for channel head advance by seepage erosion. In this case, channel incision might result from saturation overland flow draining a large area, but once incision has occurred, seepage erosion advances the channel head upslope to the point where the ground does not saturate.

**Field Site**

To test the application of the runoff and erosion thresholds, we selected a field site where extensive mapping and process studies have been conducted (figure 2). The general area, known as Tennessee Valley, lies north of San Francisco in Marin County, California, and is underlain by intensely deformed greywacke, chert, and greenstone (Wahrhaftig 1984). The ground is mostly soil-mantled, with the gravelly soil typically thin (<1 m) on side slopes and ridge tops and thickening to several meters in the colluvial fills in the valley axes. Local bedrock outcrops occur on some ridges and on steep slopes. The climate is Mediterranean with an average annual precipitation of about 760 mm (Rantz 1968). Vegetation varies with topographic position and aspect, with native and European grasses covering many of the ridgetops and gentler side slopes, and dense stands of coyote brush mixed with locally impenetrable poison oak occurring in many of the hollows and the steep side slopes in narrow canyons. The area was grazed by cattle from the early 1800s until about 1981; this grazing apparently increased the area of grass relative to brush and, when it was most heavily grazed in the late 19th century, the grass cover was probably greatly reduced relative to the present condition (Montgomery 1991).

The channel network mapped in the field (Montgomery and Dietrich 1989) is shown in figure 2. Channels extend up to and often through convergent areas of thick colluvial deposits. The channels in the steep canyons have bedrock floors. In the main valley the channel is incised into a broad, several meter thick deposit of alluvium and colluvium. Radiocarbon dating in this area (Montgomery 1991), and elsewhere in the Bay area (Reneau et al. 1990) indicates that the aggradation of colluvium and alluvium here took place during the Holocene. A variety of direct and indirect evidence, gathered from historical accounts, recovered artifacts, and aerial photographs indicates that the channel in the main valley cut down and formed a terrace about 100 yrs ago during the period of most intense grazing and that many of the tributary channels were either formed or extended into the upslope colluvium at this time (Montgomery 1991).

As reported by Montgomery and Dietrich (1988, 1989, 1992), the drainage area to the mapped channel heads systematically decreases with increasing local slope (figure 3). For any particular slope, contributing area to the observed channeled heads ranges about one order of magnitude. The data for channeled and unchanneled valleys below and above the channel head suggest that the channel heads lie at a topographic threshold (figure 3), as might be expected from the threshold theories presented above.
Figure 2. Map of the channel network (heavy solid lines), areas of thick colluvium (stippled area) and thick valley floor deposits of colluvium and alluvium (shaded area) in the Tennessee Valley study area of Marin County, California (modified from Montgomery and Dietrich 1989).

Field work in this area [Montgomery 1991] and a nearby site with similar topography, geology and vegetation [Wilson and Dietrich 1987; Wilson 1988] has demonstrated that shallow subsurface flow and saturation overland flow are the principal runoff mechanisms in response to rainstorms. By mid-winter when the ground has become sufficiently moist, storms delivering about 70 mm in about two days can cause extensive zones of saturation overland flow to form in the unchanneled and small channeled valleys. In the past few years, despite severe drought conditions, we have observed saturation overland flow extend over 100 m above the present channel heads in the lower gradient valleys. When this occurs, the water above the head typically runs very clear, carrying little or no sediment. In the channels just downstream, however, we have observed newly scoured banks and local levee deposits on the grass where the small channels (<20 cm wide) have shoaled, causing water to spill onto the surrounding surface. One intensively monitored abrupt channel head a few meters high has migrated upslope at about 20–40 cm per year for at least the past 3 yrs [Montgomery 1991]. Both seepage erosion, which leads to undermining and wall collapse, and saturation overland flow, which removes collapsed material once it falls over the head cut, are responsible for erosion.

In 1986, during monitoring of runoff at an unchanneled valley north of this site, 40 cm of rain fell in 11 days, producing saturation overland flow...
over 100 m in length (essentially to the divide). The grass was bent down by the flow and, locally at the upslope tip of the small channel and the downslope end of the valley, the overland flow had a Froude number \(>1.0\), although no obvious surface erosion occurred. Measurements of the runoff produced during this storm revealed that it had roughness properties compatible with a laminar flow description, i.e., the calculated roughness varied inversely with Reynolds number, yielding a \(K\) (as defined above) of about 10,000 (Wilson 1988, p. 109). Despite this large Reynolds number, the data gave no indication of reaching a constant roughness. We suspect that the flow was indeed turbulent, but the momentum defect caused by the vegetation resulted in a turbulent eddy diffusion coefficient that did not vary with height above the bed, giving a laminar-like flow resistance relationship.

As saturation overland flow is largely confined to convergent zones along valley axes, erosion on the surrounding hillslopes is primarily due to processes not involving surface runoff, i.e., mass-wasting processes. Shallow landslides involving just the soil-mantle are common in this area. Due to the dense brush in the steep canyon areas, it is difficult to map all the scars, even in the field. Some of these scars are probably several decades old based on the vegetation cover and sharpness of the edges, and in other cases, where bedrock is near the surface, it is difficult to define a discreet failure area. The scars are commonly only 5 to 10 m across and about twice as long. Using aerial photographs and field inspection, we mapped 39 landslide scars in the 1.2 km\(^2\) larger catchment of the two shown in figure 2. The scars are most commonly found on the steeper footslopes and at the downstream end of small steep, unchanneled valleys. Given the uncertainty in mapping these subtle features when we compare the observed and predicted pattern of landsliding, we will interpret the mapped scars as indicating areas of mass instability, rather than as discrete points of failure.

On the gentler hillslopes and on the ridges, sediment transport occurs primarily by biogenic transport associated with the burrowing activity of animals (Reneau et al. 1988; Black and Montgomery 1991). Given the minor role of water in transporting soil, downslope transport by this biogenic activity is probably largely slope dependent. Perhaps the well-defined hilltop convexities in this area, as would be expected from a slope dependent transport law (Gilbert 1909), attest to this process.

### Topographic Analysis

Prediction of the full extent of the channel network in a landscape using the above threshold theories requires digital elevation data of sufficiently high resolution that the finest scale source-area basins can be quantitatively analyzed. At least for our field area (but probably in general), the digital elevation model (DEM) from the United States Geological Survey 7.5' quadrangle was useless. Although elevations were digitized every 25 m along north-south lines in this DEM, the individual profiles were 150 m apart, and the data were interpolated to a rectangular array of data points spaced 30 m apart (Bauer and Anderson unpub. data). As shown by Bauer and Anderson and clearly visible by inspection of a topographic map generated from the data, this DEM (made available for the first time in January 1992) cannot resolve valleys with wavelengths <1000 m with much accuracy. Given the general tendency for channel networks to extend into fine-scale valleys in steep terrain (Montgomery and Dietrich 1988, 1989; Dietrich and Dunne 1993) we question whether analyses based on USGS DEM's can be used to infer channel networks or the transition from hillslope to channel-driven erosion processes.

As an alternative, we took advantage of specially flown, low-elevation high resolution black and white photographs to generate our own DEM. Several mapped ground features were used to control registration of the digital coordinates. Elevation data were obtained at a density of about every 10 m for the 1.21 km\(^2\) catchment from stereo digitization of the photographs. The low canopy cover permitted clear ground visibility, and we selected data points to capture topographic change rather than to
follow a regular grid. Elevation error for individual points we estimate to be ±0.5 m, horizontal error is about ±3 m.

For the simple steady-state threshold of erosion theories described above, any digital terrain model that enables one to compute local $a/b$ and $M$ with reasonable accuracy should be sufficient. As many have found, however, grid-based analyses so far tend to produce undesirable artifacts (e.g., Fairfield and Leymarie 1991). To avoid this problem and to analyze landscape morphology in a manner strongly tied to its hydrologic response, we have relied on the digital terrain model, TOPOG (O'Loughlin 1986; Moore et al. 1988b; Vertessy et al. 1990).

TOPOG grids the data points, smooths the data to remove sinks, draws contours and then lines roughly normal to the contours (minimum distance lines) that extend up from low elevations to ridges. These lines are approximately the flow lines for shallow subsurface and overland flow, and the combination of two adjacent flow lines intersecting two successive contours divides the land surface into distinctly shaped elements. For each element, the total contributing area, $a$, can be calculated and the ratio $a/b$ from the bottom contour length of the element. The local slope, $M$, is the sine of the gradient from the contour interval divided by the average length of the flow lines joining them in an element.

In order to link runoff and erosion processes to landscape morphology, we took advantage of the element shape created by TOPOG to classify each element into divergent, planar, and convergent morphology. We used the simple criterion that if $(b_2 - b_1)/(b_2 + b_1)$ exceeded a set percentage change, the element fell into one of the three categories. Here $b_1$ and $b_2$ are the contour length of the upslope and downslope sides of the element, respectively. This percentage change is somewhat arbitrary. We chose the smallest values estimated to be relatively free of artifacts of the model ($<-0.10$ is convergent, $>0.10$ is divergent, otherwise planar) to resolve the fine-scale topography. Figure 4 shows the spatial distribution of the three element types for our map representation of our site with 5 m contour intervals and average spacing between the “flow lines” of about 20 m. This combination of element size and criterion for shape distinction successfully shows the valley axes as convergent and the ridge lines as divergent. The relative distribution of divergent and planar elements is, however, clearly dependent on flow line spacing and contour interval relative to the cr iter-
rion. If the percentage change requirement is held constant, then with decreasing spacing between the flow lines and resultant narrowing of the elements, the planar elements spread upslope at the expense of divergent ones, leaving just the triangular elements where the flow lines terminate along the ridge line. The extent of the convergent elements is much less sensitive to the size of the element.

Land surface morphology classified in this manner captures an aspect of both the form of the surface and its likely hydrologic behavior. The contour lines create a smoothed representation of the surface, and in combination with the relatively broadly spaced flow lines this tends to emphasize the relative hydrologic convergence as compared to the local detailed shape of the ground surface. We will use element classification to guide evaluation of $T/q$, as shown below.

In order to make comparisons with our channel initiation theory, we have also classified convergent elements as channeled or unchanneled. Channeled elements were identified by tracing the observed channel network (figure 2) onto a computer-generated map of the catchment. We found that nearly all of the "channeled" elements fell within the observed range of topographic threshold for channel heads of $(a/b)S^2 = 200$ m to $(a/b)S^2 = 25$ m reported by Montgomery and Dietrich (1992) (figure 5) (here $S$ is the tan $\theta$). This excellent agreement between field-determined values of $a/b$ and tan $\theta$ and that calculated from the DEM for channeled portions of the landscape strongly supports the use of the DEM as an accurate representation of the actual ground surface. Only where the threshold relations are projected to steep slopes where channels do not occur in the field do the planar and unchanneled elements also fall within this range. We will discuss this later.

Further evidence of the reliability of the data comes from comparing the computed $a/b$ and slope data using TOPOG with that from a completely different grid-based digital terrain model developed by Bauer (pers. comm. 1992). For all elements, the average of the logarithm of $a/b$ is 58.9 m using TOPOG and 59.0 using Bauer’s model. The average slope of the elements is $23.4^\circ$ using TOPOG, whereas the average slope using Bauer’s model is $22.5^\circ$. In both cases the standard deviations were essentially identical. This suggests that artifacts peculiar to a particular digital terrain model are minimum.

Building upon the characteristic form concept popularized by Kirkby (1971), Willgoose (1989) has suggested that in a slope-area analysis of the kind in figures 2 and 5, hillslopes should show a systematic increase in contributing area with increasing slope [a convex profile], because hillslope form is presumably set by slope-dominated transport processes. Channeled portions of the landscape should show a systematic decrease in slope with increasing drainage area because of the importance of water runoff as well as slope in the transport process [a concave profile], and the length of the hillslope would be reflected in a distinctive inflection point on an area-slope or slope-area graph of elements derived from a digital terrain model. Tarboton et al. (1991) have subsequently claimed to observe this inflection point in their analysis of digital elevation data, but this may be an artifact of their analysis of low resolution USGS DEM’s. Furthermore, an innovative theoretical analysis based on stability theory by Loewenherz (1991a, 1991b) suggests that the channel head may extend upslope of the inflection point. While there is a clear change in the contributing area-slope relation associated with the transition from divergent to convergent elements (figure 5), there is no simple inflection in our data.
Threshold Analysis

The thresholds of saturation (equation 2), slope stability (equation 4), and erosion by saturation overland flow (equations 10 and 11) are functions of $a/b$ and $M$; hence the derived threshold equations can be readily compared with the observed distribution of element types. This comparison can place significant constraints on the appropriateness of the threshold and, in effect, permits a rational approach for evaluating the parameters in each of the threshold theories. In fact, given the assumption of steady-state runoff, such a comparison may be the only reasonable way to parameterize fully these theories.

Threshold of Saturation. All three thresholds are controlled at least partly by the ratio of transmissivity to runoff rate ($T/q$). In the threshold of saturation (equation 2) $T/q$ is the slope of the line relating $a/b$ to $M$. There are at least three ways to assess $T/q$. Field measurements, such as pump tests on piezometers, can provide estimates of $T$, and $q$ can be selected from precipitation records. Given the high spatial variability of $T$ and the uncertainty of what exactly $q$ is (because of the steady-state assumption), this method of assessing $T/q$ cannot be used. Based on an intensive hydro-logic study near our field site (Wilson 1988), we estimate the transmissivity to be about 17 m$^2$/d and, surprisingly, not to vary significantly with topographic position (hollow versus ridge) because of the dominance of the high conductivity of the surface soil. This estimate must still be considered extremely crude, but without some other constraint, we have no rational way of choosing a significant steady-state $q$ to drive the erosion thresholds.

Another method, which at least would help evaluate the general validity of the model, would be to map in a watershed the farthest upslope extent of saturated areas developed from a large precipitation event and determine whether this position has a similar value of $a/(bM)$ throughout the basin, as expected from equation (2). We have attempted to do this, but because of an extended drought in California, we have had very little opportunity to make observations. It is also difficult to locate field observations with the precision necessary to identify which element it belongs to in the digital terrain model. The last significant overland flow event was in March of 1989 after a roughly 0.08 m rainfall. For six different hollows, it appears the $a/(bM)$ value for the farthest upslope extent of surface saturation ranged from 200 to at least 2000 m, with four of the points apparently falling between 500 to 1000 m. If $T$ is roughly 17 m$^2$/d, then $q$ in this case would be 0.034 to 0.068 m/d. No landslides occurred during this event, but some of the smallest upslope channel tips did scour, and a carefully monitored channel head advanced about 0.4 m upslope.

A third method to estimate $T/q$, and one that seems most appropriate for a general threshold of erosion investigation, is to use the landscape morphology to place constraints on the possible value of $T/q$. Based on both field observations and what we would expect from simple physical intuition, we would not expect the divergent elements, which are almost exclusively found on the ridges, to experience saturation. Furthermore, the lower gradient convergent elements making up channeled and unchanneled valleys ought to become saturated during events large enough to cause landsliding or channelization by saturation overland flow. Figure 6 shows the saturation threshold line for three values of $T/q$ plotted against the various element types. Clearly, a value of about 350 m for $T/q$ causes nearly all divergent elements to be unsaturated while nearly all of the convergent and channeled ones are saturated (figure 6) [see Dietrich et al. [1992] for comparison with individual data points]. The vast majority of the planar elements are also predicted to be unsaturated, which seems reasonable. This value of $T/q$ would require a $q$ of 0.05 m/d if $T$ is 17 m$^2$/d. To judge whether an 0.05 m/d roughly steady-state runoff is possible, we used the simple analytical model of Iida (1984) to estimate that it would take roughly 9 days for a 59 m long planar slope (approximate mean slope length here) to reach steady-state with a transmissivity of 17 m$^2$/d and rainfall of 0.05 m/d. A storm about this magnitude (0.40 m) and duration occurred near here in 1986 [Wilson 1988]. The maximum precipitation recorded in 10 days at a rain gage in this general area is 1.1 m (California Dept. Water Resources 1981). Figure 7 shows the extent of ground water saturation for steady-state runoff and a $T/q$ of 350 m. The saturated region, with few exceptions, is confined to the valley network of channeled and unchanneled convergent topography. Some of the long triangular saturated patches along the divide are clearly artifacts of the model, but otherwise the predicted saturated patches appears consistent with field observations.

Although we cannot conclude that we know with any precision an appropriate value of $T/q$, the above analyses place significant constraint on it. The estimated value of 350 m is not unreasonable, based on field observations. In the analysis to follow, we will take $T/q$ as fixed and evaluate the
Figure 6. Comparison of threshold theories with digital terrain data of the Tennessee Valley catchment modified from Dietrich et al. 1992.

Slope Stability Threshold. Given the assignment of $T/d$ to a value of 350 m, the slope stability threshold (equation 4) has two remaining parameters, $\phi$ and $C$. The ratio of wet soil bulk density to water content. Previous studies have suggested that for the coarse-textured soils in the field area, the angle of internal friction $\phi$ can be as high as 45° and is likely to lie between 40 and 45°. These soils are generally cohesionless although the roots of the grass and cowpea brush would add a slight apparent cohesion. This strength contribution has been ignored here. In Figure 6, we show the position of the slope stability threshold for the possible range of $\phi$ values and a large number of planar elements to be unstable, a prediction not consistent with our field observations, in which most of the current landslide scars are found in convergent areas, some in planar, and none in divergent ones. Hence, this lower bound appears to be too low. Very few elements...
Figure 9. Comparison of observed channel network with: (A) the empirical topographic threshold based on field observations at the channel head (modified from Montgomery and Dietrich 1992); (B) the pattern of areas predicted to be subject to shallow landsiding.

tion strongly suggests that the tributary channels are not driven by instability originating at the junction with a higher order channel junction. A threshold theory for channel initiation should presumably also explain why channels are discontinuous. The second feature we note is that only the elements in the narrow bedrock canyons of the steeper tributaries have $(a/b)S^2$ values in excess of the channel head threshold range. Why don't higher-order channels well downslope from the channel head also have such high values? Is this an artifact of the insensitivity of the analysis? Field
observations suggest otherwise. As shown in figure 2, the lower gradient valleys in the field area contain thick deposits of alluvium and colluvium, whereas the narrow bedrock canyons show little or no evidence of stored sediment. Analysis of landuse and landscape change in this area (Montgomery 1991) indicates that the main low gradient channel network into which the steeper uplands drain only entrenched into its floodplain after introduction of intensive cattle grazing. Based on observations nearby in one of the only watersheds never subject to cattle grazing, it is reasonable to suggest that the low gradient channels may not have been continuous before disturbance. Our abil-
ity to analyze this problem is limited because the elements in the low gradient valleys are quite large and the local hydraulics within an element associated with a channel are likely to be quite different than that estimated from the element as a whole. Nonetheless, the field observations and topographic analysis suggest a tendency for the low gradient valleys to aggrade with poorly defined channels when unaltered by landuse.

The third and fourth features of figure 9A have to do with the unchanneled elements within or near the threshold range of channel initiation. While none of the channel heads extend below the \(|a/b|S^2\) of 25 m, many elements lying above this value are unchanneled. Most of these elements form distinct linear patches on the steep side slopes of the bedrock canyons and lie in subtle hollows. Empirically we would argue that these elements lie in areas prone to channelization, but either because of local variations in such properties as saturated conductivity and friction angle, or, and more likely, because of a tendency for diffusive sediment transport to fill in channels here the sites lack mappable channels. If these elements lie above the erosion threshold, the latter interpretation seems most plausible. The fourth feature, and one that Montgomery and Dietrich (1992) emphasized, is the tendency for elements with the value of \(|a/b|S^2\) just below 25 m to lie at the topographic transition from valley to ridge. Montgomery and Dietrich (1992) argue that this strongly suggests that ridge and valley development and ultimately their spacing is influenced by the threshold of channel initiation. To be consistent, the erosion theories described above should not predict surface instability upslope of these elements.

Figure 9B shows the pattern of elements predicted to be unstable for a \(T_i/q\) of 350 m and a \(\phi\) of 40° and their relationship to the channel network. As discussed above, these parameters are reasonably well constrained by topographic analysis and observations; nonetheless, there remains significant uncertainty about the full extent of the potentially unstable ground. The comparison with the channel network shows that nearly half of the channel heads lie in or just below areas predicted to be above the slope stability threshold. The vast majority of the unstable elements, however, do not drain directly to a channel head nor lie on the path of a channel. Hence, while landsliding can contribute to channel head advance upslope, the tendency for landsliding alone is not sufficient to maintain a channel. On the other hand, many of these unchanneled, but unstable elements do correspond to the elements either within the channeled range of \(|a/b|S^2\) or just below it (compare with figure 9A). These observations indicate that the linear patches of unchanneled elements within the channel head range shown in figure 9A correspond to unstable areas that just lack sufficient runoff to maintain a channel.

In figure 9C, the pattern of relative sediment transport capacity \((\tau_i - \tau_c)^{1.5}\) is mapped and compared with the observed channel network. Although channel incision is expected to occur once a critical boundary shear stress is exceeded (and possibly supercritical flow occurs), if the divergence of the boundary shear stress and thereby sediment transport field is negative downslope, aggradation and smothering of any channelization tendency is likely. In comparing the excess relative sediment transport capacity with the observed channel network it is worth remembering how approximate the parameters are in the boundary shear stress model and the large scale of the element relative to the size of the channel. Nonetheless, the similarity throughout the basin of results for 35 different tributaries for a single set of parameters supports the mechanistic interpretation proposed here.

In most of the tributaries, the channel head lies slightly downslope of the point where the boundary shear stress exceeds the critical value for saturation overland flow erosion. The other channel heads generally correspond to sites where saturation overland flow erosion predominates. Relative sediment transport increases downstream along the channel network, but for all nine discontinuous tributaries, the downstream end of the channel corresponds to the reach where the relative sediment transport declines downslope. A decline in the downslope direction also occurs along the network where the channels leave the narrow canyons and enter the broad valley, in agreement with the historical tendency for deposition to occur in the lower valley and possibly have only a poorly formed channel. This seems to provide strong support for the general structure of our model.

Most of the unchanneled elements that lie within the channel head range of \(|a/b|S^2\) and along the valley axes are predicted to be unstable due to saturation overland flow erosion (compare 9C,A). Although this is good correspondence between the threshold theory and the observed topographic threshold, it does not explain why these elements remained unchanneled. One simple answer, which presumes the model to be reasonably accurate, is that the assumed critical boundary shear stress and roughness are slightly incorrect for application of this model to the field observations. Figure 10
shows the effect of changing the critical boundary shear stress by only a factor of two. Because of the third power dependency on critical boundary shear stress in the threshold model, a huge difference in the extent of channel network (elements above the critical boundary shear stress) is predicted with small changes in \( \tau_c \). This implies that the difference between observed and predicted channel network in figure 9C is explicable by a small modification of \( \tau_c \). Unfortunately, for the purposes of field verification of this erosion threshold model, this result suggests that critical boundary shear stress is an important parameter which is virtually unknowable at the level necessary to test model predictions. While it will be useful to determine whether \( \tau_c \) is 1000 or 100 dyne/cm\(^2\) from field experiments, we will likely not be able to characterize it for an entire basin to within 50%. Hence, like saturated conductivity or transmissivity, critical boundary shear stress, even though it has a smaller range of values, may not be a parameter that can be sufficiently well-known from just field observations and an "effective" value must be estimated from response of the system as a whole.

If we accept the general validity of the saturation overland flow erosion threshold model, then our model results clearly demonstrate that changes in land use or climate that alter the surface resistance to erosion should have a dramatic effect on the stability of the channel network. Cattle grazing in this area reduced the critical boundary shear stress and caused significant expansion of the channel network (figure 10).

Figure 9D summarizes the spatial significance of different erosion processes predicted from the threshold theories. It shows the ridge lines dominated by diffusive (slope-dependent) sediment transport processes that feed sediment to landslidel-prone planar and subtle convergent slopes bordering the ridges and valley sides. Surface runoff in the larger valleys then conveys the sediment further downstream. Mapping of process dominance, as shown in figure 9D, should prove useful in catchment sediment budget studies. A more formidable task is the development of sediment transport laws supported with field observations that permit routing of sediment down hillslopes and along the channel network.

**Conclusions**

With the development of digital terrain models capable of analyzing large sets of digital elevations, it is now possible both to quantify the detailed topography of landscapes and to explore process-based models in explaining mappable features. One of the most useful features to map is the channel network. We have shown that it can be used to place significant constraint on erosion theories. Parameterization of such theories may be quite difficult because of the high spatial variability of the controlling parameters and the sensitivity of the model outcome to their values. Instead of relying on field observations alone for evaluation of the parameters, the topography and the extent of the observed channel network itself can be used to determine the parameters. But once such a calibration is done, application elsewhere is needed to validate the model.

Despite this limitation, the comparison between the threshold theory of saturation, erosion by saturation overland flow, and threshold of slope instability and the observed topography and extent of the channel network in our field area suggests several conclusions. The upslope extent of the channel network is controlled by a combination of slope instability and erosion by saturation overland flow. These two thresholds have very different \( a/b \) to \( M \) relationships and should not be represented by a single function, as some may choose when examining the empirical data. Although seepage erosion may also be important, our analysis suggests that it is not required. While slope instability contributes to channel head advance, it alone does not control the tendency for channelization: overland flow appears to be required. Hence, there are many steep areas bordering the channeled valleys that cross the topographic threshold for channelization, but remain unchanneled because only shallow landsliding occurs. In other, steeper environments with greater relief, perhaps debris flow scour from landsliding can act to cut channel-like features, thus reducing the importance of overland flow. Application of our model to locations in Oregon and Washington support this interpretation.

The threshold of erosion by saturation overland flow varies as the third power of critical boundary shear stress. Consequently, the extent of the channel network incised by erosive overland flow varies dramatically with changes in surface resistance. This helps explain the widespread gullying in the grazing land surrounding the San Francisco Bay and probably many other areas where overland flow is important in channel incision.

Application of the models proposed here to other landscapes with similar erosion processes will require high resolution digital elevation models to capture the topographic control on runoff and erosion mechanisms. The models proposed here should be quite general to landscapes where
Figure 10. Area predicted to be eroded by saturation overland flow for a critical boundary shear stress of (A) 320 dyne/cm² and (B) 160 dyne/cm². This pattern corresponds to the two threshold lines shown in figure 6.
shallow subsurface and saturation overland flow runoff mechanisms predominate. For model evaluation, detailed mapping of the channel network and location of landslide scars is essential. Such data are rarely available at present.

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