ABSTRACT

We propose a graphical technique to analyze the entirety of landforms in a catchment to define quantitatively the spatial variation in the dominance of different erosion processes. High-resolution digital elevation data of a 1.2 km\(^2\) hilly area where the channel network had been mapped in the field were used in the digital terrain model, TOPOG, to test threshold theories for erosion. The land surface was divided into \(\sim 20 \text{ m}^2\) elements whose shapes were then classified as convergent, planar, or divergent. The entire landscape plotted on a graph of area per unit contour length against surface gradient shows each planform plotting as a separate field. A simple steady-state hydrologic model was used to predict zones of saturation and areas of high pore pressure to mimic the extreme hydrologic events responsible for erosive instability of the land surface. The field observation that saturation overland flow is rare outside convergent zones provided a significant constraint on the hydrologic parameter in the model. This model was used in threshold theories to predict areas of slope instability and areas subject to erosion by saturation overland flow, both of which can contribute to channel initiation. The proportion of convergent elements predicted to exceed the threshold varies greatly with relatively small changes in surface resistance, demonstrating a high sensitivity to land use such as cattle grazing. Overall, the landscape can be divided, using erosion threshold lines, into areas prone to channel instability due to runoff and stable areas where diffusive transport predominates.

INTRODUCTION

Although numerical models can create realistic-looking landscapes by generating ridge and valley topography (e.g., Ahnert, 1976; Kirkby, 1987; Howard, 1990; Willgoose et al., 1991), the three-dimensional form of real landscapes and the processes shaping them are, surprisingly, not well quantified. The recent development of digital terrain models, however, permits quantitative analysis of actual landscapes and thus provides an opportunity to examine the relation between sediment transport processes and landscape form (e.g., Moore et al., 1988a, 1988b; Vertessy et al., 1990; Tarboton et al., 1991).

Herein we propose a graphical technique for characterizing real landscapes using digital elevation data to examine the application of erosion theories. We focus on a landscape where previous studies (Montgomery and Dietrich, 1988, 1989, 1992; Montgomery, 1991) have indicated that there is good evidence that threshold-based erosion models are appropriate. Field monitoring and mapping suggest that surface erosion leading to channel initiation occurs where there is a resistance to saturation overland flow (see Dunne, 1980), seepage erosion (see Dunne, 1990), or shallow landsliding is exceeded. These processes predominate in valleys, whereas on ridges the shallow soil is currently transported primarily by biogenic activity such as burrowing by gophers (Black and Montgomery, 1991), a process that perhaps can be treated as largely slope dependent. Although our analysis focuses on current runoff and erosion processes, our results give insight into how erosion would vary with land use and climate change as well as what erosion laws appear appropriate for modeling long-term landscape evolution.

THRESHOLD THEORIES

We use three simple threshold equations to explore the relation between landform and current erosion processes. These equations predict a threshold condition for ground saturation, a threshold to landslide instability of the ground due to high pore pressures, and a threshold of erosion due to saturation overland flow. All three theories have been proposed in various forms by others. Here we write them in a form useful for analysis using a digital terrain model. Several simplifying assumptions are made to reduce the parameters to those that can be crudely estimated from field data and the proposed graphical analysis. These theories, however, are at the same general level of simplicity as those in most numerical models of landscape evolution. Specifically, we use a steady-state runoff model that assumes that runoff occurs as subsurface flow parallel to the ground surface during significant hydrologic events and that saturated conductivity and transmissivity of the soil mantle are spatially constant. (Despite large differences in soil thickness between ridges and unchanneled valleys, the saturated conductivity and, consequently, the transmissivity are dominated by highly conductive near-surface soil [Wilson, 1988; Montgomery, 1991].) We propose that steady-state runoff for an extreme event will mimic the spatial variation in surface saturation and overland flow that would occur in natural transient storm events responsible for saturation overland flow erosion and landsliding. We also assume that the vegetation and soil properties controlling surface resistance to erosion are spatially constant.

For steady-state, shallow subsurface runoff parallel to the ground surface, the ground will be saturated if the precipitation (minus evaporation and deeper drainage), \(q\), times the area of the upslope catchment, \(a\), equals or exceeds the maximum flux the surface layer can conduct, computed from the product of transmissivity, \(T\), surface slope, \(M\), and unit length of the contour across which the catchment is draining, \(b\) (O'Loughlin, 1986). Because of the steep slopes in the study area, \(M\) is calculated as the more physically correct sine of the ground-surface inclination, \(\theta\), rather than \(\tan \theta\), as used by O'Loughlin (1986). The threshold of ground saturation can be expressed as:

\[
\frac{a}{b} > \frac{T}{q} = M.
\]  

This simple hydrologic model has been used with good success to predict saturated zones and runoff response in the computer model TOPOG by O'Loughlin (1986) and by Moore et al. (1988a, 1988b). It is essentially the same model that underlies the widely used TOPMODEL by Beven and Kirkby (1979).

All parts of the landscape where the area per unit contour length, \(a/b\), equals or exceeds the term on the right-hand side of equation 1 will be saturated. Note that if measured values from a landscape of \(a/b\) are
plotted against $M$, then equation 1 will be a straight line with a slope given by the hydrologic parameters $T/q$, and all points above this line will be saturated. This observation suggests that a useful analysis of digital elevation data is to divide the land surface into discrete small catchments for which the physical attributes $a/b$ and $M$ can be determined and plotted on such a graph. Other threshold criteria can be expressed as functions of these two physical characteristics.

A coupled hydrologic and slope stability model proposed by Dietrich et al. (1986) and subsequently modified and tested by Montgomery and Dietrich (1989) can be written as:

$$\frac{a}{b} \geq 2 \left(1 - \frac{\tan \theta}{\tan \phi}\right) \frac{T}{Mq}.$$  \hspace{1cm} (2)

Slope instability occurs where $a/b$ equals or exceeds the term on the right-hand side, which varies with the ratio of ground surface, $\tan \theta$, to angle of internal friction, $\tan \phi$. Note that the hydrologic component of equation 2 is the same model as that which leads to equation 1 and that equation 2 uses a form of the infinite slope model that ignores strength contribution due to cohesion; i.e., at failure $\theta = [(\rho_s - \rho_w m)/(\rho_s + \rho_w) \phi]$.

Several authors have suggested that channel initiation by overland flow can be estimated by assuming that incision occurs where some critical boundary shear stress, $\tau_c$, or some other measure of resistance is exceeded (e.g., Horton, 1945; Schaefer, 1979; Moore et al., 1988b; Vertessy et al., 1990; Montgomery, 1991). In the steady-state model used here, saturation overland flow discharge is simply equal to $qa - TMb$; i.e., water that cannot be carried as shallow subsurface flow must travel overland. This equation can be solved for the discharge that attains sufficient depth for a given slope to produce a boundary shear stress equal to the critical value for the surface. Letting $qa - TMb = udb$, $r_c = \rho_w M$, $u = (2gdM)^{0.5} (f)^{-5}$, and $f = Kv/ud$, the following threshold of erosion equation can be derived in the desired form:

$$\frac{a}{b} \geq \frac{\alpha}{qM^2} + \frac{T}{qM}.$$  \hspace{1cm} (3)

Here $\alpha = 2 \times 10^{-4} \tau_c^3 K^{-1}$ and $\tau_c$ is the critical boundary shear stress, $K$ is the roughness intercept for the inverse relation between friction factor, $f$, and Reynolds number (velocity, $u$, times depth, $d$, divided by the kinematic viscosity, $v$) that typifies laminar-like flow in grasslands (Dunne and Dietrich, 1980; Wilson, 1988; see review in Reid, 1989). Gravitational acceleration is $g$, and the numerical constant is for $10^\circ$C water and has units of (cm-s)$^{-5}$/g$^3$.

**DIGITAL TERRAIN MODEL ANALYSIS**

We selected an area in the hilly grass and chaparral lands north of San Francisco where extensive mapping and hydrologic studies have been conducted (Wilson and Dietrich, 1987; Montgomery and Dietrich, 1988, 1989; Black and Montgomery, 1991; Montgomery, 1991). These studies have shown that saturation overland flow is common in the lower-gradient valleys, and most of the larger debris-flow scars originate at channel heads. Digital elevation data were obtained at a density of about every 10 m for the 1.21 km$^2$ catchment from stereo digitization of low-level black and white photographs.
white aerial photographs; several mapped ground features were used to control the registration of digital coordinates. Taking advantage of the clear ground visibility, we selected data points to capture topographic change rather than to follow a regular grid. The digital elevation model component of TOPOG (O’Loughlin, 1986) was then used to construct digital surfaces. A second program in TOPOG divides the surface by drawing the equivalent of flow lines across the contours from valley bottoms to divides at a user-specified interval (see O’Loughlin, 1986, for examples). The program draws flow lines starting at low elevations and projects upslope, so contour length separating flow lines tend to narrow on topographically divergent slopes. Individual elements defined by a pair of contour lines on the upslope and downslope sides of the element and a pair of streamlines on the lateral boundaries (Fig. 1) are thus created.

For each element, the total contributing area, $a$, can be calculated and the ratio $a/b$ determined from the bottom contour length of the element. The local slope between the two contour lines making up the element is also determined. Each element shape was classified as convergent, planar, or divergent, according to the difference in length of the upslope, $b_1$, and downslope contour length, $b_2$, of the element; i.e., whether the ratio $(b_2 - b_1)/(b_2 + b_1)$ exceeded a set percentage change. This percentage is somewhat arbitrary. We chose the smallest values estimated to be relatively free of artifacts of the model ($<-0.10$ is convergent, $>0.10$ is divergent, otherwise planar). These values clearly delineated the convergent valley axes in the landscape (Fig. 1). In addition, we used detailed field mapping of the current extent of the channel network to classify those elements (always convergent) that contained a channel.

Figure 2 shows the data field for each element type as a function of specific catchment ($a/b$) and local slope ($\tan \theta$) for a 5 m contour interval and 20 m interval between flow lines. For a given slope, the channel elements drain the largest specific catchment, whereas the divergent elements drain the smallest. There is very little overlap between divergent and convergent element data fields and essentially none between divergent and channel elements. These differences are much larger than that which might be created by the definition of element types. Comparison of the data fields for different contour intervals and flow-line spacing shows that there are some artifacts in Figure 2. The apparent log-linear data in the divergent elements are a portion of the triangular-shaped elements created at divides by diverging flow lines. They have no physical relation to each other, and although they are purely an artifact of the analysis, they still describe aspects of the divergent topography and so were retained. For planar elements, the minimum size of the elements varies with slope as controlled by the contour spacing (5 m in this case), hence $a/b$ (minimum) = 5/\tan \theta, and no points plot below this value in Figure 2. Reducing the contour spacing and the flow-line spacing, however, had negligible effect on the...
general distribution of the data. Furthermore, we found that a grid-based
digital terrain model developed by one of us (Bauer) also produced very
similar results.

LANDFORM AND PROCESS THRESHOLDS

In order to apply the threshold equations 1-3, the parameters controlling
their predictions must be estimated. This problem is more constrained
than may at first be apparent. We know from field work in this area during
major runoff events (Wilson and Dietrich, 1987; Montgomery, 1991) that
extensive saturation overland flow does not occur on divergent and planar
slopes. We have seen exceptions, but they are of local extent only. This
allows us to vary the hydrologic ratio, \( T/q \), and note where the line of
saturation crosses the various topographic elements (see thresholds box
and then comparison with data in other boxes; Fig. 2). Note that \( T/q \) of
350 m clips just the top of the divergent and planar elements. Conversely,
we would expect most of the channel elements to be saturated during an
extreme event, and, as expected, the line with \( T/q \) of 350 m lies below
most of the elements; halving this value adds fewer channel elements
but cuts deeply across the divergent element field. We have observed
saturation overland flow extend up to nearly the divide in this area in
common storms; it is therefore realistic to predict that nearly all of the
convergent elements will saturate in a major storm. On the basis of pre-
vious studies (Wilson, 1988), the transmissivity is largely controlled at
saturation by the high conductivity of the near-surface soils, apparently
differing little between nose and hollow. We estimate this value to be 17
m²/d, which indicates that for a \( T/q \) of 350 m, the \( q \) is equal to 5 cm/d of
precipitation or runoff. According to the analytical procedure of Iida
(1984), it would take about 9 d for a 59-m-long planar slope to reach
steady state with this estimated transmissivity and rainfall; a storm of about
this magnitude (45 cm) and duration occurred here in 1986 (Wilson,
1988).

With \( T/q \) defined, the only remaining parameter in equation 2 is the
angle of internal friction; previous studies indicate that this value may
commonly be as high as 45° (Reneau et al., 1984). This stability analysis
ignores the contribution of apparent cohesion from root strength. In Figure
2, the slope stability threshold line is plotted for values of \( \phi \) equal to 35°
and 45°. The number of elements predicted to fail is greatly increased with
the diminished friction angle, including many divergent elements. This is
inconsistent with field observations in which shallow soil landslides are
mostly in convergent zones. Decreasing \( T/q \) tends to overpredict the
number of planar elements in which failure might be expected. Prelimi-
nary field mapping indicates that all but 5 of the 39 shallow landslide scars
currently visible in the study area lie above the predicted threshold line for
a \( T/q \) of 350 m and \( \phi = 40° \). Hence, the position of the slope stability
curve is reasonably well defined. This curve lies on the outer edge of the
data fields, suggesting that this stability criterion imposes a physical limit
on hillslope morphology, as traditionally argued from simpler analyses
(i.e., Strahler, 1950). The steeper channel and convergent elements lie
above the threshold line of slope stability; this is consistent with field
observations.

The saturation overland flow erosion threshold given in equation 3
requires estimates of \( K \), \( \tau_c \), and \( q \) for a specified \( T/q \) and water tempera-
ture (which sets the value of \( \nu \)). Measurements at times of significant
overland flow at a nearby grassland site document that \( K \) is about 10000
for well-vegetated surfaces (Wilson, 1988, p. 109) and that shear stresses
in excess of 200 dyne/cm² generated by this flow did not cause measur-
able incision into the vegetated areas. High runoff events, however,
have caused scour in the unvegetated tips of the channel networks. Reid (1989),
in a thorough review of critical shear stress values, showed that \( \tau_c \) must
exceed 1000 dyne/cm² to incise a well-vegetated mat, whereas values of
250 to 500 dyne/cm² are sufficient to incise the underlying soils. If \( \tau_c \)
were 1000 dyne/cm², for a threshold line defined by equation 3 to sepa-
rate channeled from unchanneled elements, \( q \) would have to be ~1.5 m/d,
clearly an impossibility. As argued by Reid (1989), channel initiation in
grasslands is likely to occur where local barren areas caused by fire,
trampling, or other effects reduce the critical shear stress to the underlying
value of the soil.

Figure 2 shows the position of threshold lines for a \( \tau_c \) of 160
dyne/cm² and 320 dyne/cm² for \( q \) of 5 cm/d (appropriate for \( T/q = 350 \)
and \( T = 17.5 m²/d \)). Given the observed roughness value, \( \tau_c \) and \( q \)
are constrained by the threshold relation required to separate channeled
from unchanneled elements. The critical shear stress cannot be signifi-
cantly different from 160 to 320 dyne/cm² without requiring either too
great or too small a precipitation rate. Note that, as indicated in equation 3,
a doubling of critical boundary shear stress is equivalent to decreasing
precipitation or \( K \) by a factor of 8. Such a doubling greatly alters the location
of this erosion threshold line and raises questions concerning the worth of
trying to measure a parameter in the field, the value of which is so difficult
to estimate but so greatly affects the pattern of channelization. This analy-
sis also shows, however, how land use practices or climate change that
only modestly alter surface resistance \( (\tau_c) \) could lead to a great expansion
of the channel network into previously unchanneled elements. Such an
expansion due to the introduction of cattle grazing and presumed reduc-
sion in surface resistance has been proposed for this area by Montgomery
and Dietrich (1989) and Montgomery (1991). The inverse dependence of
the erosion threshold on precipitation is also consistent with the proposal
by Reneau et al. (1986) and Montgomery (1991) that drying and warming
of this area from the Pleistocene into the Holocene caused channel heads
to retreat downslope.

In Figure 3, stability fields generated by the combination of equations
1 through 3 are labeled with the dominant hydrologic and erosion proc-
esses. Simple field observations constrain the threshold of saturation and
slopes, stability lines and lead to the interpretation that a significant proportion of the landscape, nearly all the divergent elements, most of the planar, and some of the convergent ones, are currently stable or resistant to surface erosion due to runoff. In this region below the erosion threshold lines (shaded region in Fig. 3), surface transport is currently dominated by the slope-driven (diffusive) processes of soil creep, rain splash, and biogenic disturbance. The area labeled "SOF Without Erosion" mostly includes elements lying in unchanneled valleys currently collecting colluvium (but which in the long term undergo periodic erosion, apparently driven by climatic change [Reneau et al., 1986]). Hence, this graph shows a new way to analyze the entirety of landscapes to define quantitatively the spatial variation in the dominance of different erosion processes in a catchment. An analysis of this kind should also be a useful tool in constructing sediment budgets for catchments, when the spatial distribution of dominant transport processes must be ascertained (i.e., Dietrich and Dunne, 1978; Dietrich et al., 1982; Reid, 1989).

CONCLUSIONS
The plotting of discrete elements of landscapes separated by planform on a graph of the two topographic variables that control runoff and erosion (area per unit contour width and surface gradient) provides a powerful means to evaluate hydrologic and erosional hypotheses about real landscapes. This analysis and the parallel study reported by Montgomery and Dietrich (1992) on the relation between position of the channel head and landscape scale add considerable support to the Horton hypothesis of a threshold control on surface instability leading to channel development (Horton, 1945). The simple, steady-state, hydrologic models employed here could be made more realistic by using a transient model, and this introduces more parameters, and at least for the problem of examining the relations between form and erosion thresholds, it is not clear that the transient model is essential to the outcome. The threshold theories are similar to those used to model landscape evolution; the analysis performed here suggests a procedure to evaluate transport laws used in numerical models. The plotting of threshold lines on a data field of a/b vs. M is an instructive procedure for examining the possible effects of land use and climatic change.

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