Allotaxonometry

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Principles of Complex Systems, Vols. 1 & 2
CSYS/MATH 300 and 303, 2021–2022| @pocsvox

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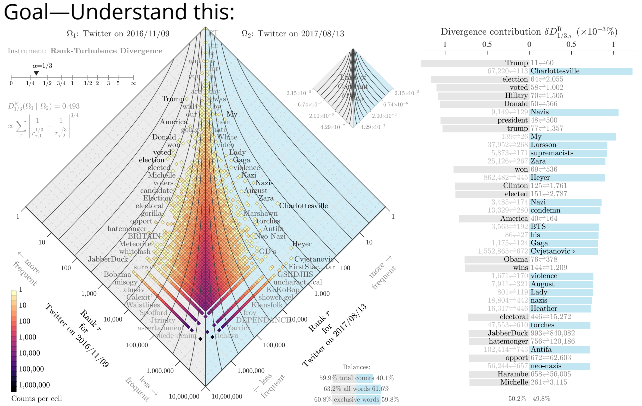
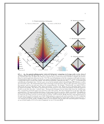
Outline

Goal—Understand this:

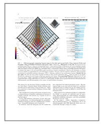


Site (papers, examples, code):
http://compstorylab.org/allotaxonometry/

Foundational papers:



"Allotaxonometry and rank-turbulence divergence: A universal instrument for comparing complex systems"
Dodds et al.,
, 2020. [5]



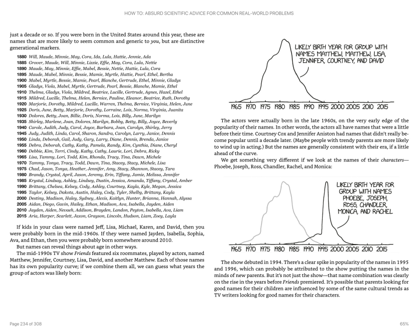
"Probability-turbulence divergence: A tunable allotaxonometric instrument for comparing heavy-tailed categorical distributions"
Dodds et al.,
, 2020. [6]

Basic science = Describe + Explain:

- Dashboards of single scale instruments helps us understand, monitor, and control systems.
- Archetype: Cockpit dashboard for flying a plane
- Okay if comprehendible.
- Complex systems present two problems for dashboards:
  1. Scale with internal diversity of components: We need meters for every species, every company, every word.
  2. Tracking change: We need to re-arrange meters on the fly.
- Goal—Create comprehendible, dynamically-adjusting, differential dashboards showing two pieces:[1]
  1. 'Big picture' map-like overview,
  2. A tunable ranking of components.

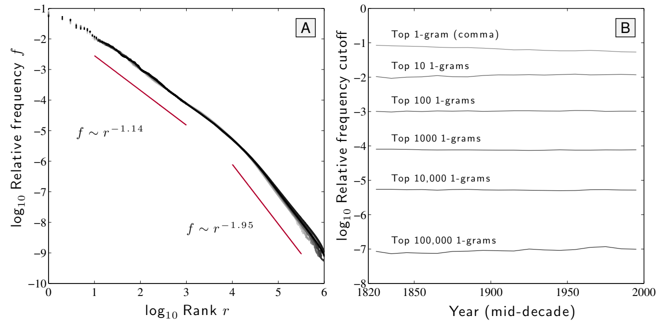
[1] See the lexicocalorimeter

Baby names, much studied: [12]



How to build a dynamical dashboard that helps sort through a massive number of interconnected time series?

"Is language evolution grinding to a halt? The scaling of lexical turbulence in English fiction suggests it is not"
Pechenick, Danforth, Dodds, Alshaabi, Adams, Dewhurst, Reagan, Danforth, Reagan, and Danforth.
Journal of Computational Science, **21**, 24–37, 2017. [14]



For language, Zipf's law has two scaling regimes: [18]

$$f \sim \begin{cases} r^{-\alpha} \text{ for } r \ll r_{\mathrm{b}}, \\ r^{-\alpha'} \text{ for } r \gg r_{\mathrm{b}}, \end{cases}$$

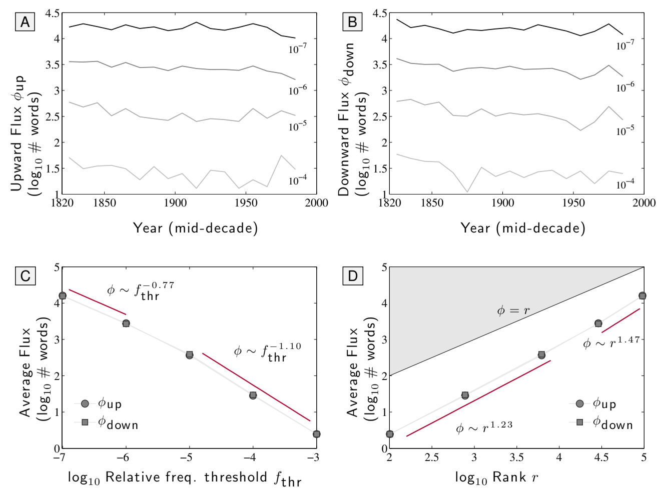
When comparing two texts, define Lexical turbulence as flux of words across a frequency threshold:

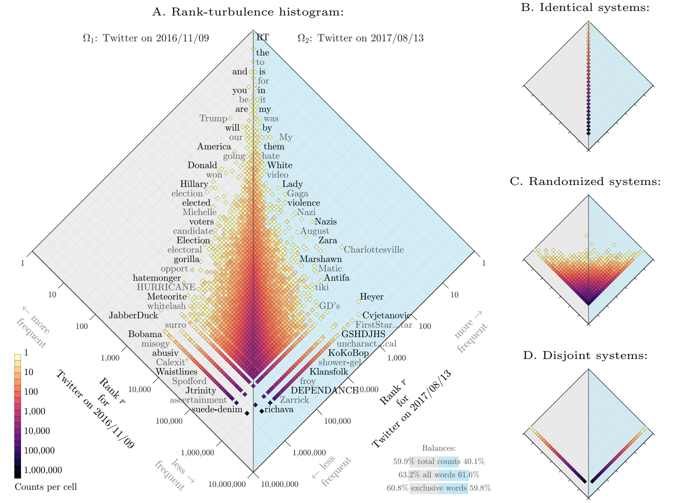
$$\phi \sim \begin{cases} f_{\mathrm{thr}}^{-\mu} \text{ for } f_{\mathrm{thr}} \ll f_{\mathrm{b}}, \\ f_{\mathrm{thr}}^{-\mu'} \text{ for } f_{\mathrm{thr}} \gg f_{\mathrm{b}}, \end{cases}$$

Estimates: $\mu \simeq 0.77$ and $\mu' \simeq 1.10$, and f_{b} is the scaling break point.

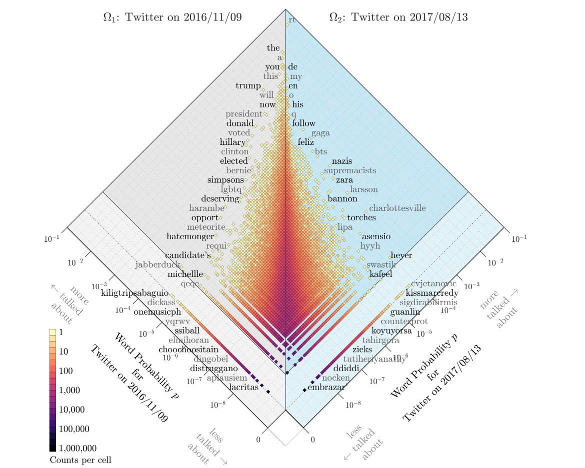
$$\phi \sim \begin{cases} r^{\nu} = r^{\alpha\mu'} \text{ for } r \ll r_{\mathrm{b}}, \\ r^{\nu'} = r^{\alpha'\mu} \text{ for } r \gg r_{\mathrm{b}}. \end{cases}$$

Estimates: Lower and upper exponents $\nu \simeq 1.23$ and $\nu' \simeq 1.47$.

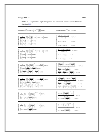




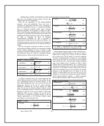
Zipf-turbulence histogram for probability:



So, so many ways to compare probability distributions:



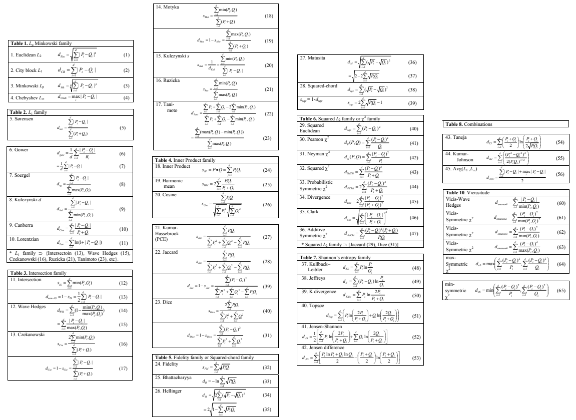
"Families of Alpha- Beta- and Gamma-Divergences: Flexible and Robust Measures of Similarities"
Cichocki and Amari,
Entropy, **12**, 1532-1568, 2010. [2]



"Comprehensive survey on distance/similarity measures between probability density functions"
Sung-Hyuk Cha,
International Journal of Mathematical Models and Methods in Applied Sciences, **1**, 300–307, 2007. [1]

- Comparisons are distances, divergences, similarities, inner products, fidelities ...
- A worry: Subsampled distributions with very heavy tails
- 60ish kinds of comparisons grouped into 10 families

Quite the festival:



- We want two main things:
  1. A measure of difference between systems
  2. A way of sorting which types/species/words contribute to that difference
- For sorting, many comparisons give the same ordering.
- A few basic building blocks:
  - $|P_i - Q_i|$ (dominant)
  - $\max(P_i, Q_i)$
  - $\min(P_i, Q_i)$
  - $P_i Q_i$
  - $|P_i^{1/2} - Q_i^{1/2}|$ (Hellinger)

Table 1. L_p Minkowski family

1. Euclidean L_2	$d_{Euc} = \sqrt{\sum_{i=1}^{d} \lvert P_i - Q_i \rvert^2}$	(1)
2. City block L_1	$d_{CB} = \sum_{i=1}^{d} \lvert P_i - Q_i \rvert$	(2)
3. Minkowski L_p	$d_{Mk} = \sqrt[p]{\sum_{i=1}^{d} \lvert P_i - Q_i \rvert^p}$	(3)
4. Chebyshev L_∞	$d_{Cheb} = \max_i \lvert P_i - Q_i \rvert$	(4)

Table 2. L_1 family

5. Sørensen	$d_{sor} = \dfrac{\sum_{i=1}^{d} \lvert P_i - Q_i \rvert}{\sum_{i=1}^{d} (P_i + Q_i)}$	(5)
6. Gower	$d_{gow} = \dfrac{1}{d} \sum_{i=1}^{d} \dfrac{\lvert P_i - Q_i \rvert}{R_i}$	(6)
	$= \dfrac{1}{d} \sum_{i=1}^{d} \lvert P_i - Q_i \rvert$	(7)
7. Soergel	$d_{sg} = \dfrac{\sum_{i=1}^{d} \lvert P_i - Q_i \rvert}{\sum_{i=1}^{d} \max(P_i, Q_i)}$	(8)
8. Kulczynski d	$d_{kul} = \dfrac{\sum_{i=1}^{d} \lvert P_i - Q_i \rvert}{\sum_{i=1}^{d} \min(P_i, Q_i)}$	(9)
9. Canberra	$d_{Can} = \sum_{i=1}^{d} \dfrac{\lvert P_i - Q_i \rvert}{P_i + Q_i}$	(10)
10. Lorentzian	$d_{Lor} = \sum_{i=1}^{d} \ln(1 + \lvert P_i - Q_i \rvert)$	(11)

* L_1 family $\supset$ {Intersection (13), Wave Hedges (15), Czekanowski (16), Ruzicka (21), Tanimoto (23), etc}.

- Information theoretic sortings are more opaque
- No tunability

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- Shannon's Entropy:

$$H(P) = \langle \log_2 \frac{1}{p_\tau} \rangle = \sum_{\tau \in R_{1,2;\alpha}} p_\tau \log_2 \frac{1}{p_\tau} \quad (1)$$

- Kullback-Liebler (KL) divergence:

$$D^{\mathrm{KL}}(P_2 \,\|\, P_1) = \left\langle \log_2 \frac{1}{p_{2,\tau}} - \log_2 \frac{1}{p_{1,\tau}} \right\rangle_{P_2}$$

$$= \sum_{\tau \in R_{1,2;\alpha}} p_{2,\tau} \left[\log_2 \frac{1}{p_{2,\tau}} - \log_2 \frac{1}{p_{1,\tau}} \right]$$

$$= \sum_{\tau \in R_{1,2;\alpha}} p_{2,\tau} \log_2 \frac{p_{1,\tau}}{p_{2,\tau}}. \quad (2)$$

- Problem: If just one component type in system 2 is not present in system 1, KL divergence = ∞.
- Solution: If we can't compare a spork and a platypus directly, we create a fictional spork-platypus hybrid.
- New problem: Re-read solution.

- Jensen-Shannon divergence (JSD): [9, 7, 13, 1]

$$D^{\mathrm{JS}}(P_1 \,\|\, P_2)$$

$$= \tfrac{1}{2} D^{\mathrm{KL}}\left(P_1 \,\middle\|\, \tfrac{1}{2}[P_1 + P_2]\right) + \tfrac{1}{2} D^{\mathrm{KL}}\left(P_2 \,\middle\|\, \tfrac{1}{2}[P_1 + P_2]\right)$$

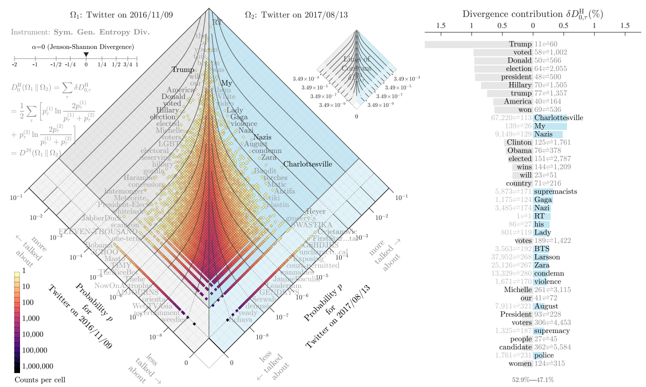
$$= \frac{1}{2} \sum_{\tau \in R_{1,2;\alpha}} \left(p_{1,\tau} \log_2 \frac{p_{1,\tau}}{\frac{1}{2}[p_{1,\tau} + p_{2,\tau}]} + p_{2,\tau} \log_2 \frac{p_{2,\tau}}{\frac{1}{2}[p_{1,\tau} + p_{2,\tau}]} \right). \quad (3)$$

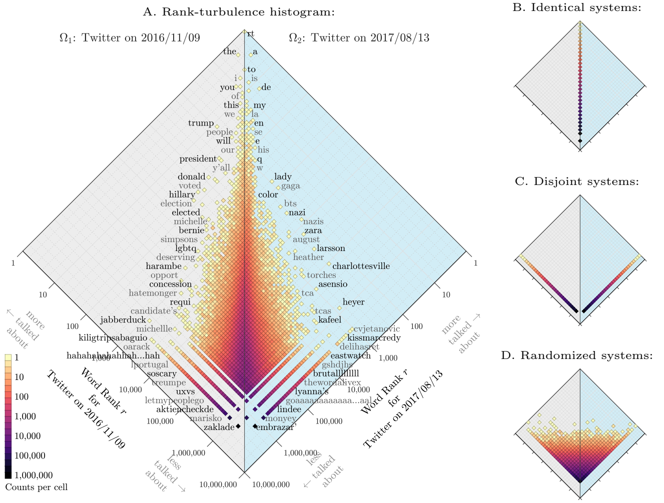
- Involving a third intermediate averaged system means JSD is now finite: $0 \le D^{\mathrm{JS}}(P_1 \,\|\, P_2) \le 1$.
- Generalized entropy divergence: [2]

$$D^{\mathrm{AS2}}_{\alpha}(P_1 \,\|\, P_2) =$$

$$\frac{1}{\alpha(\alpha - 1)} \sum_{\tau \in R_{1,2;\alpha}} \left[\left(p_{\tau,1}^{1-\alpha} + p_{\tau,2}^{1-\alpha}\right) \left(\frac{p_{\tau,1} + p_{\tau,2}}{2}\right)^{\alpha} - (p_{\tau,1} + p_{\tau,2}) \right]. \quad (4)$$

Produces JSD when $\alpha \to 0$.





Exclusive types:

- We call types that are present in one system only 'exclusive types'.
- When warranted, we will use expressions of the form $\Omega^{(1)}$-exclusive and $\Omega^{(2)}$-exclusive to indicate to which system an exclusive type belongs.

Desirable rank-turbulence divergence features:

1. Rank-based.
2. Symmetric.
3. Semi-positive: $D^{\mathsf{R}}_{\alpha}(\Omega_1 \,||\, \Omega_2) \geq 0$.
4. Linearly separable, for interpretability.
5. Subsystem applicable: Ranked lists of any principled subset may be equally well compared (e.g., hashtags on Twitter, stock prices of a certain sector, etc.).
6. Zipfophilic: Able to handle systems with rank-ordered component size distribution that are heavy-tailed.
7. Scalable: Allow for sensible comparisons across system sizes.
8. Tunable.
9. Story-finding: Features 1–8 combine to show which component types are most 'important'

Some good things about ranks:

- Working with ranks is intuitive
- Affords some powerful statistics (e.g., Spearman's rank correlation coefficient)
- Can be used to generalize beyond systems with probabilities

A start:

$$\left|\frac{1}{r_{\tau,1}} - \frac{1}{r_{\tau,2}}\right|. \qquad (5)$$

- Inverse of rank gives an increasing measure of 'importance'
- High rank means closer to rank 1
- We assign tied ranks for components of equal 'size'
- Issue: Biases toward high rank components

We introduce a tuning parameter:

$$\left|\frac{1}{[r_{\tau,1}]^{\alpha}} - \frac{1}{[r_{\tau,2}]^{\alpha}}\right|^{1/\alpha}. \qquad (6)$$

- As $\alpha \to 0$, high ranked components are increasingly dampened
- For words in texts, for example, the weight of common words and rare words move increasingly closer together.
- As $\alpha \to \infty$, high rank components will dominate.
- For texts, the contributions of rare words will vanish.

Trouble:

- The limit of $\alpha \to 0$ does not behave well for

$$\left|\frac{1}{[r_{\tau,1}]^{\alpha}} - \frac{1}{[r_{\tau,2}]^{\alpha}}\right|^{1/\alpha}.$$

- The leading order term is:

$$\left(1-\delta_{r_{\tau,1} r_{\tau,2}}\right) \alpha^{1/\alpha} \left|\ln\frac{r_{\tau,1}}{r_{\tau,2}}\right|^{1/\alpha}, \qquad (7)$$

which heads toward ∞ as $\alpha \to 0$.

- Oops.
- But the insides look nutritious:

$$\left|\ln\frac{r_{\tau,1}}{r_{\tau,2}}\right|$$

is a nicely interpretable log-ratio of ranks.

Some reworking:

$$\delta D^{\mathsf{R}}_{\alpha,\tau}(R_1 \,||\, R_2) \propto \frac{\alpha+1}{\alpha}\left|\frac{1}{[r_{\tau,1}]^{\alpha}} - \frac{1}{[r_{\tau,2}]^{\alpha}}\right|^{1/(\alpha+1)}. \qquad (8)$$

- Keeps the core structure.
- Large α limit remains the same.
- $\alpha \to 0$ limit now returns log-ratio of ranks.
- Next: Sum over τ to get divergence.
- Still have an option for normalization.

Rank-turbulence divergence:

$$D^{\mathsf{R}}_{\alpha}(R_1 \,||\, R_2) = \frac{1}{\mathcal{N}_{1,2;\alpha}} \sum_{\tau \in R_{1,2;\alpha}} \delta D^{\mathsf{R}}_{\alpha,\tau}(R_1 \,||\, R_2) \qquad (9)$$

Normalization:

- Take a data-driven rather than analytic approach to determining $\mathcal{N}_{1,2;\alpha}$.
- Compute $\mathcal{N}_{1,2;\alpha}$ by taking the two systems to be disjoint while maintaining their underlying Zipf distributions.
- Ensures: $0 \leq D^{\mathsf{R}}_{\alpha}(R_1 \,\|\, R_2) \leq 1$
- Limits of 0 and 1 correspond to the two systems having identical and disjoint Zipf distributions.

Rank-turbulence divergence:

Summing over all types, dividing by a normalization prefactor $\mathcal{N}_{1,2;\alpha}$ we have our prototype:

$$D^{\mathsf{R}}_{\alpha}(R_1 \,||\, R_2) = \frac{1}{\mathcal{N}_{1,2;\alpha}} \frac{\alpha+1}{\alpha} \sum_{\tau \in R_{1,2;\alpha}} \left|\frac{1}{[r_{\tau,1}]^{\alpha}} - \frac{1}{[r_{\tau,2}]^{\alpha}}\right|^{1/(\alpha+1)}. \qquad (10)$$

General normalization:

- Iif the Zipf distributions are disjoint, then in $\Omega^{(1)}$'s merged ranking, the rank of all $\Omega^{(2)}$ types will be $r = N_1 + \frac{1}{2}N_2$, where N_1 and N_2 are the number of distinct types in each system.
- Similarly, $\Omega^{(2)}$'s merged ranking will have all of $\Omega^{(1)}$'s types in last place with rank $r = N_2 + \frac{1}{2}N_1$.
- The normalization is then:

$$\mathcal{N}_{1,2;\alpha} = \frac{\alpha+1}{\alpha} \sum_{\tau \in R_1} \left| \frac{1}{[r_{\tau,1}]^{\alpha}} - \frac{1}{[N_1 + \frac{1}{2}N_2]^{\alpha}} \right|^{1/(\alpha+1)} + \frac{\alpha+1}{\alpha} \sum_{\tau \in R_1} \left| \frac{1}{[N_2 + \frac{1}{2}N_1]^{\alpha}} - \frac{1}{[r_{\tau,2}]^{\alpha}} \right|^{1/(\alpha+1)}. \tag{11}$$

Limit of $\alpha \to 0$:

$$D^{\mathsf{R}}_{0}(R_1 \,\|\, R_2) = \sum_{\tau \in R_{1,2;\alpha}} \delta D^{\mathsf{R}}_{0,\tau} = \frac{1}{\mathcal{N}_{1,2;0}} \sum_{\tau \in R_{1,2;\alpha}} \left| \ln \frac{r_{\tau,1}}{r_{\tau,2}} \right|, \tag{12}$$

where

$$\mathcal{N}_{1,2;0} = \sum_{\tau \in R_1} \left| \ln \frac{r_{\tau,1}}{N_1 + \frac{1}{2}N_2} \right| + \sum_{\tau \in R_2} \left| \ln \frac{r_{\tau,2}}{\frac{1}{2}N_1 + N_2} \right|. \tag{13}$$

- Largest rank ratios dominate.

Limit of $\alpha \to \infty$:

$$D^{\mathsf{R}}_{\infty}(R_1 \,\|\, R_2) = \sum_{\tau \in R_{1,2;\alpha}} \delta D^{\mathsf{R}}_{\infty,\tau} = \frac{1}{\mathcal{N}_{1,2;\infty}} \sum_{\tau \in R_{1,2;\alpha}} \left(1 - \delta_{r_{\tau,1} r_{\tau,2}}\right) \max_{\tau} \left\{ \frac{1}{r_{\tau,1}}, \frac{1}{r_{\tau,2}} \right\}. \tag{14}$$

where

$$\mathcal{N}_{1,2;\infty} = \sum_{\tau \in R_1} \frac{1}{r_{\tau,1}} + \sum_{\tau \in R_2} \frac{1}{r_{\tau,2}}. \tag{15}$$

- Highest ranks dominate.

Probability-turbulence divergence:

$$D^{\mathsf{P}}_{\alpha}(P_1 \,\|\, P_2) = \frac{1}{\mathcal{N}^{\mathsf{P}}_{1,2;\alpha}} \frac{\alpha+1}{\alpha} \sum_{\tau \in R_{1,2;\alpha}} \left| \left[p_{\tau,1}\right]^{\alpha} - \left[p_{\tau,2}\right]^{\alpha} \right|^{1/(\alpha+1)}. \tag{16}$$

- For the unnormalized version ($\mathcal{N}^{\mathsf{P}}_{1,2;\alpha}$=1), some troubles return with 0 probabilities and $\alpha \to 0$.
- Weep not: $\mathcal{N}^{\mathsf{P}}_{1,2;\alpha}$ will save the day.

Normalization:

With no matching types, the probability of a type present in one system is zero in the other, and the sum can be split between the two systems' types:

$$\mathcal{N}^{\mathsf{P}}_{1,2;\alpha} = \frac{\alpha+1}{\alpha} \sum_{\tau \in R_1} \left[p_{\tau,1}\right]^{\alpha/(\alpha+1)} + \frac{\alpha+1}{\alpha} \sum_{\tau \in R_2} \left[p_{\tau,2}\right]^{\alpha/(\alpha+1)} \tag{17}$$

Limit of α=0 for probability-turbulence divergence

- if both $p_{\tau,1} > 0$ and $p_{\tau,2} > 0$ then

$$\lim_{\alpha \to 0} \frac{\alpha+1}{\alpha} \left| \left[p_{\tau,1}\right]^{\alpha} - \left[p_{\tau,2}\right]^{\alpha} \right|^{1/(\alpha+1)} = \left| \ln \frac{p_{\tau,2}}{p_{\tau,1}} \right|. \tag{18}$$

- But if $p_{\tau,1} = 0$ or $p_{\tau,2} = 0$, limit diverges as $1/\alpha$.

Limit of α=0 for probability-turbulence divergence

- Normalization:

$$\mathcal{N}^{\mathsf{P}}_{1,2;\alpha} \to \frac{1}{\alpha}\left(N_1 + N_2\right). \tag{19}$$

- Because the normalization also diverges as $1/\alpha$, the divergence will be zero when there are no exclusive types and non-zero when there are exclusive types.

Combine these cases into a single expression:

$$D^{\mathsf{P}}_{0}(P_1 \,\|\, P_2) = \frac{1}{(N_1 + N_2)} \sum_{\tau \in R_{1,2;0}} \left(\delta_{p_{\tau,1},0} + \delta_{0,p_{\tau,2}}\right). \tag{20}$$

- The term $\left(\delta_{p_{\tau,1},0} + \delta_{0,p_{\tau,2}}\right)$ returns 1 if either $p_{\tau,1} = 0$ or $p_{\tau,2} = 0$, and 0 otherwise when both $p_{\tau,1} > 0$ and $p_{\tau,2} > 0$.
- Ratio of types that are exclusive to one system relative to the total possible such types,

Type contribution ordering for the limit of α=0

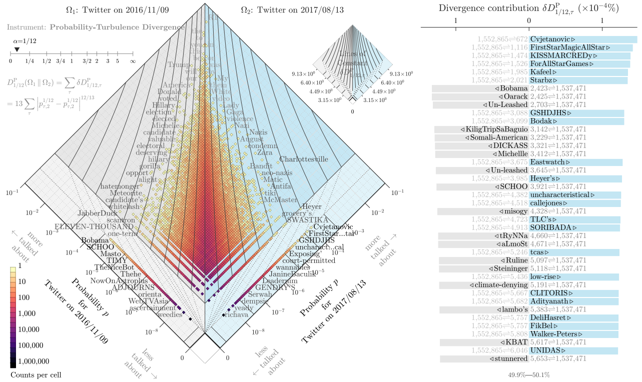
- In terms of contribution to the divergence score, all exclusive types supply a weight of $1/(N_1 + N_2)$. We can order them by preserving their ordering as $\alpha \to 0$, which amounts to ordering by descending probability in the system in which they appear.
- And while types that appear in both systems make no contribution to $D^{\mathsf{P}}_{0}(P_1 \,\|\, P_2)$, we can still order them according to the log ratio of their probabilities.
- The overall ordering of types by divergence contribution for α=0 is then: (1) exclusive types by descending probability and then (2) types appearing in both systems by descending log ratio.

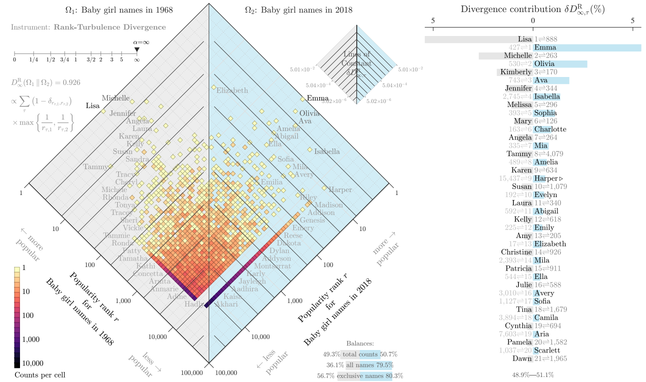
Limit of $\alpha=\infty$ for probability-turbulence divergence

$$D^{\mathrm{P}}_{\infty}(P_1 \,\|\, P_2) = \frac{1}{2} \sum_{\tau \in R_{1,2;\infty}} \left(1 - \delta_{p_{\tau,1},p_{\tau,2}}\right) \max\left(p_{\tau,1}, p_{\tau,2}\right) \tag{21}$$

where

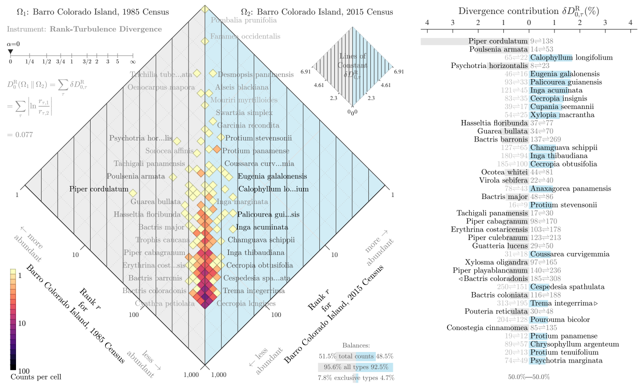
$$\mathcal{N}^{\mathrm{P}}_{1,2;\infty} = \sum_{\tau \in R_{1,2;\infty}} \left(p_{\tau,1} + p_{\tau,2}\right) = 1 + 1 = 2. \tag{22}$$

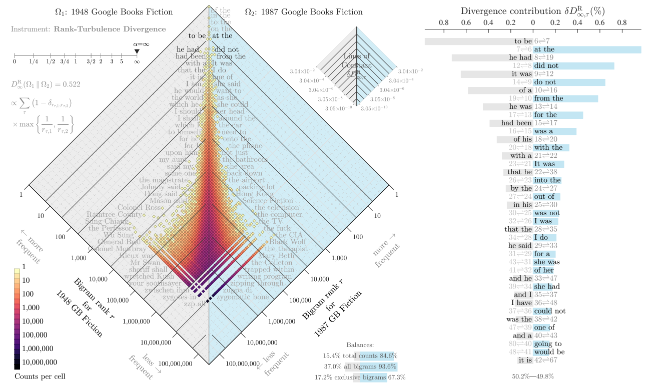


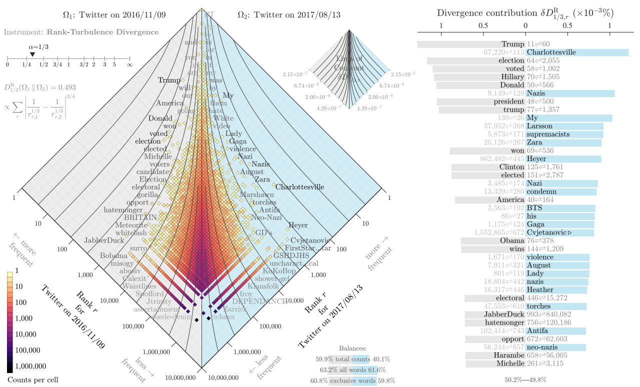


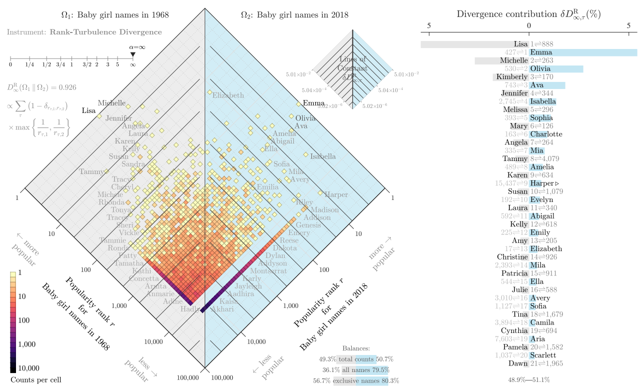
Connections for PTD:

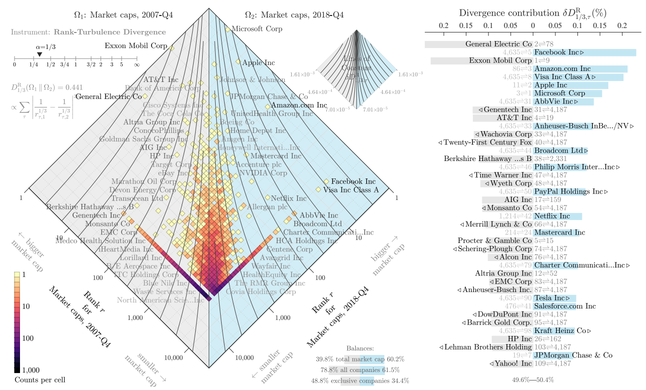
- $\alpha = 0$: Similarity measure Sørensen-Dice coefficient [4, 16, 10], F_1 score of a test's accuracy [17, 15].
- $\alpha = 1/2$: Hellinger distance [8] and Mautusita distance [11].
- $\alpha = 1$: Many including all $L^{(p)}$-norm type constructions.
- $\alpha = \infty$: Motyka distance [3].











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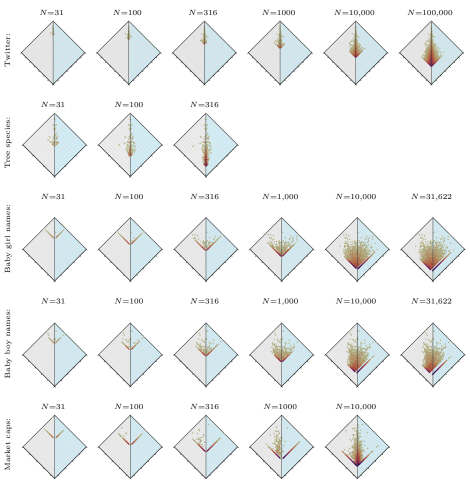

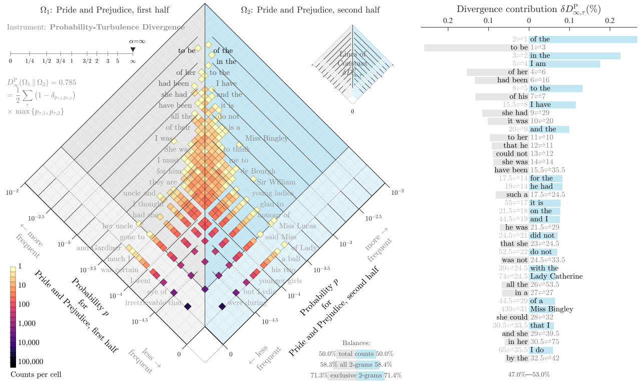

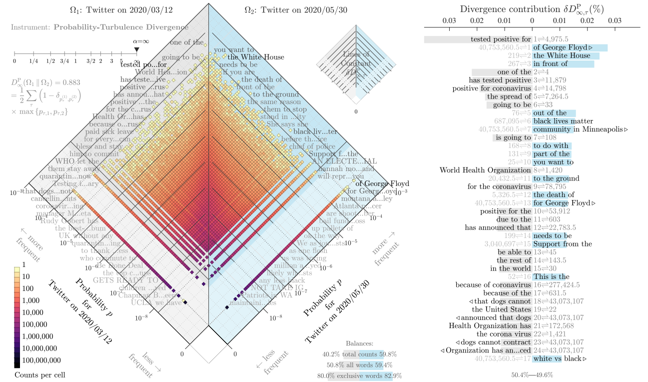

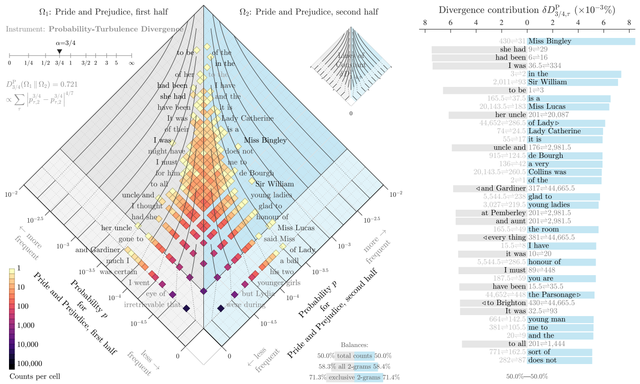

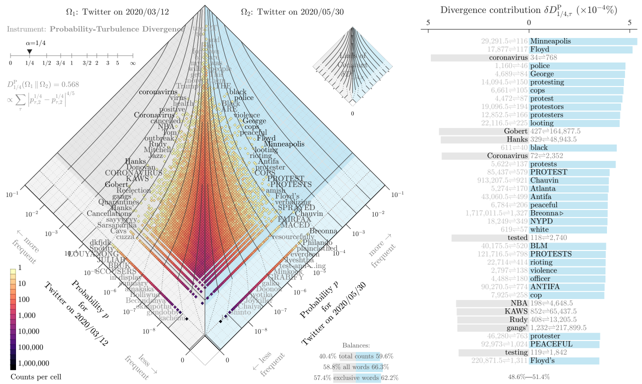

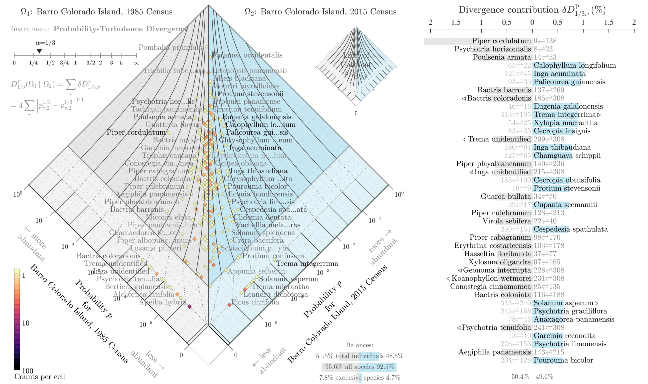

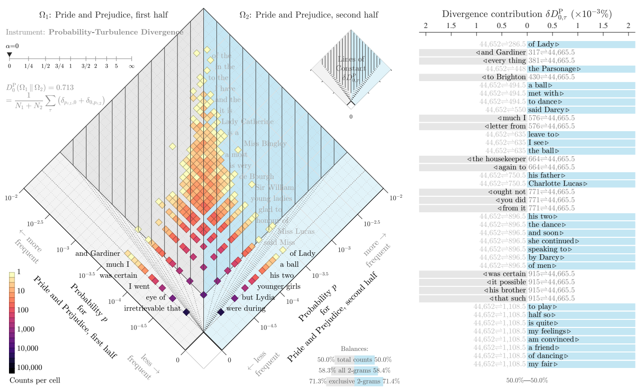

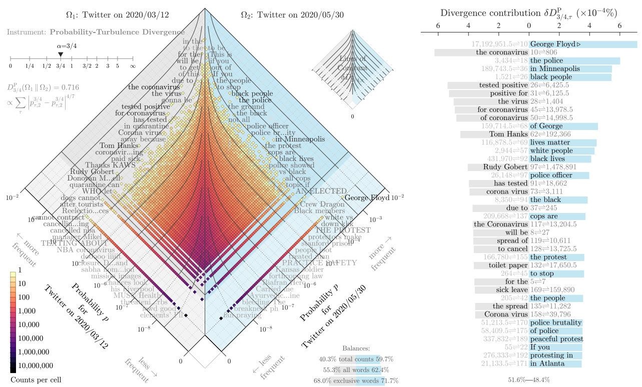

Flipbooks:

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  instrument-flipbook-3-gen-entropy-div.pdf
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  instrument-flipbook-9-google-books-trigrams-rank-div.pdf

Flipbooks:

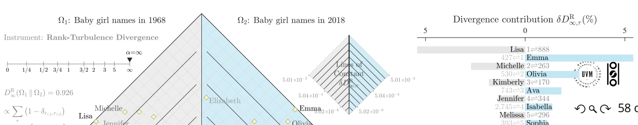
Pride and Prejudice, 1-grams
Pride and Prejudice, 2-grams
Pride and Prejudice, 3-grams
Twitter, 1-grams
Twitter, 2-grams
Twitter, 3-grams
Barro Colorado Island

Code:

https://gitlab.com/compstorylab/allotaxonometer

Claims, exaggerations, reminders:

- Needed for comparing large-scale complex systems:
  Comprehendible, dynamically-adjusting, differential dashboards
- Many measures seem poorly motivated and largely unexamined (e.g., JSD)
- Of value: Combining big-picture maps with ranked lists
- Maybe one day: Online tunable version of rank-turbulence divergence (plus many other instruments)



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