Social welfare function—A graph or curve showing the all the possible combinations of individual utilities where social welfare is the same. The social welfare function is based on given preferences, technology and resource endowment, plus some specific ethical assumption about the fair distribution of goods among consumers.

Utility possibilities frontier—A graph or curve showing the maximum possible utility of one consumer given the utility of the other consumer. It shows all the possible Pareto-efficient combinations of utilities, given the preferences of each consumer.

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Rawls, John. 1971. *A Theory of Justice*. Cambridge, MA: Harvard University Press.

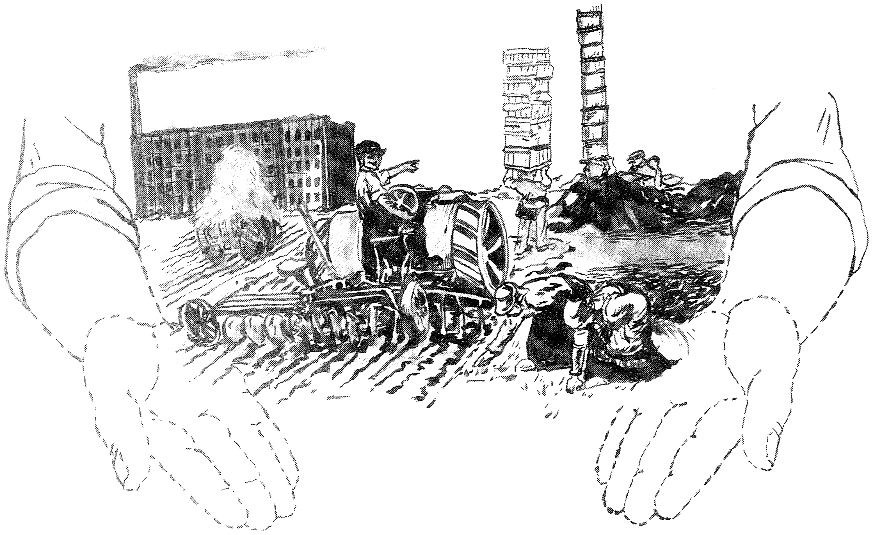
4

INTRODUCING PRICES

Perfect Competition and Pareto Efficiency

> As every individual, therefore, endeavours as much as he can both to employ his capital in the support of domestick industry, and so to direct that industry that its produce may be of the greatest value; every individual necessarily labours to render the annual revenue of the society as great as he can. He generally, indeed, neither intends to promote the publick interest, nor knows how much he is promoting it. By preferring the support of domestick to that of foreign industry, he intends only his own security; and by directing that industry in such a manner as its produce may be of the greatest value, he intends only his own gain, and he is in this, as in many other cases, led by an invisible hand to promote an end which was no part of his intention.
>
> —*Adam Smith,* An Inquiry into the Nature and Causes of the Wealth of Nations *[1776], edited by R. H. Campbell and A. S. Skinner (New York: Liberty Press, 1981, IV.ii), 456*

The Walrasian representation of a barter economy presented in chapters 1–3 is the core of contemporary microeconomic theory. Interestingly, although microeconomics is sometimes called "price theory," prices play no independent role in the basic Walrasian system. In a "frictionless" economy populated by independent consumers and producers with perfect information about prices, the results of free exchange will exactly duplicate the outcome obtained in a face-to-face barter system. The price of each good contains all the information necessary to compare its desirability to the desirability of every other good. In such an economy the "invisible hand" of the market will ensure the most efficient allocation of society's scarce resources.



STEP ONE: MAXIMIZING UTILITY SUBJECT TO A BUDGET CONSTRAINT

Let us begin by adding prices to the consumer choice model in Chapter 1. Let us give the consumer a budget (call it M for "money") to spend and an array of goods (with prices) to choose from. How does a consumer allocate his or her income among various consumer goods so as to maximize the utility the consumer receives from these goods?

Given a budget M and the prices of goods X (P_x) and Y (P_Y), how does a consumer decide how much of each good to buy? As shown in Figure 4.1, this consumer's utility is maximized at point 1 where the indifference curve I2 is just tangent to the budget line showing all the possible combinations of the goods X and Y the consumer could buy given the consumer's income the relative prices of the goods. At point 1, the consumer buys ten units of each good. Given the budget line, the consumer could have chosen point 2 but can increase his or her utility by moving to point 1 instead (on the higher indifference curve I2). Any point on the higher indifference curve I3 would be preferred to point 1, but those choices are unavailable given the budget constraint.

Chapter 1 showed that the slope of the indifference curve is equal to the marginal rate of substitution ($MRS_{Y \text{ for } X}$), which is equal to the ratio of the marginal utilities of the two goods. The equation for the budget constraint is $M = P_X X + P_Y Y$. Rearranging terms to express the budget in terms of goods X and Y yields:

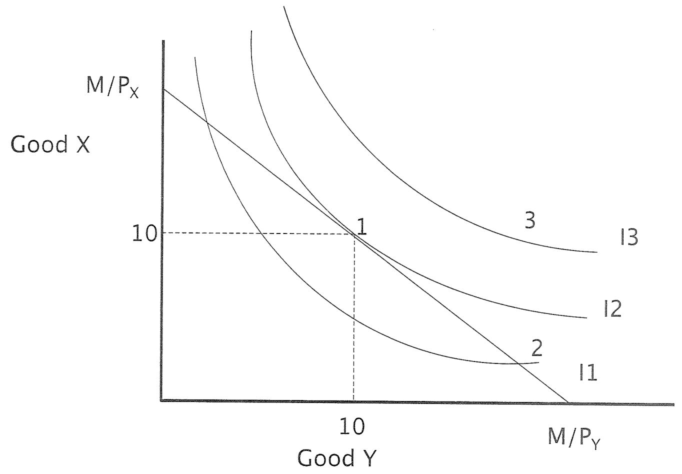


Figure 4.1. Maximizing utility subject to a budget constraint

$$(4.1) \qquad X = M/P_X - (P_Y/P_X)Y$$

The intercept M/P_X shows how many units of good X could be purchased if the consumer spent his or her entire budget on good X. Equation (4.1) shows that the slope of the budget line is equal to the price ratio of the two goods:

$$(4.2) \qquad dX/dY = -P_Y/P_X$$

So the condition for utility maximization, that the indifference curve is just tangent to the budget line, is:

$$(4.3) \qquad P_Y/P_X = MRS_{Y \text{ for } X} = MU_Y/MU_X$$

We can obtain this result using the mathematics of constrained optimization. The Lagrangian equation (see the appendix at the end of this chapter) for maximizing utility subject to a budget constraint is:

$$(4.4) \qquad Z = U(X, Y) - \lambda(P_X X + P_Y Y - M)$$

Taking the partial derivatives of this equation and setting them to zero yields:

$$\partial Z / \partial X = \partial U / \partial X - \lambda P_X = 0 \tag{4.5}$$

$$\partial Z / \partial Y = \partial U / \partial Y - \lambda P_Y = 0 \tag{4.6}$$

$$\partial Z / \partial \lambda = M - P_X X - P_Y Y = 0 \tag{4.7}$$

Moving the second term in the first two equations to the right-hand side and dividing the first equation by the second gives us the **first-order condition** for maximizing utility subject to a budget constraint:

$$(\partial U / \partial X) / (\partial U / \partial Y) = P_X / P_Y \tag{4.8}$$

As demonstrated in Figure 4.1, the ratio of marginal utilities of the two goods must equal their price ratio. To ensure that this condition maximizes utility (rather than minimizing it) we need the **second-order condition**:

$$\begin{aligned} d^2U / dX^2 = \partial^2 U / \partial X^2 + 2\,(\partial^2 U / \partial X \partial \delta Y)(-P_X / P_Y) \\ + (\partial^2 U / \partial Y^2)\, P_X^2 < 0 \end{aligned} \tag{4.9}$$

It can also be shown that in equilibrium $\lambda = (\partial U / \partial X) / P_X = (\partial U / \partial Y) / P_Y$, that is, the Lagrangian multiplier equals the marginal utility of each good divided by its price. This means that, when the consumer is maximizing utility subject to a budget constraint, the consumer gets the same utility from a dollar spent on each good. So λ can be interpreted as the **marginal utility of money.**

STEP TWO: INPUTS, OUTPUT, AND MARGINAL COST

Let us now turn to the production side and reexamine the **production possibilities frontier** by adding the costs of production to our model. We saw in Chapter 3 that the slope of the production possibilities frontier indicates the **rate of product transformation,** that is, the rate at which the resources used in the production of one good can be shifted to the production of another good while maintaining the greatest output efficiency possible. In our simple model if we produce one fewer unit of good X, then this frees up capital and labor that can be used to produce good Y. For simplicity, assume that labor is the only productive input. Suppose that producing one more unit of good Y requires two units of labor and that producing one more unit of good X requires one unit of labor. So the marginal cost of producing Y is two and the marginal cost of X is one. As depicted in Figure 4.2, the ratio of marginal costs MC_Y / MC_X (two in this example) is also the rate of product transformation of X into Y. If we give up one unit of Y this frees up enough resources (two units of labor in our simple example) to produce two units of X.

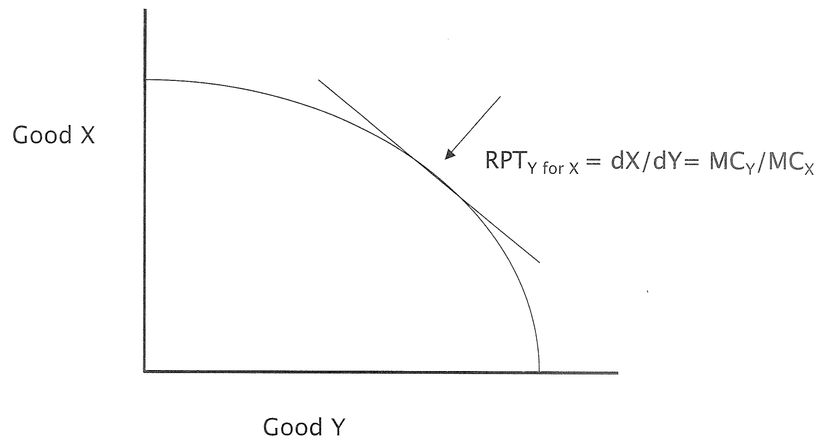


Figure 4.2. Marginal costs and the production possibilities frontier

$$RPT_{Y \text{ for } X} = MC_Y / MC_X \tag{4.10}$$

STEP THREE: PERFECT COMPETITION

The last piece of economic theory we need is the centerpiece of neoclassical microeconomics, the model of "pure" or "perfect" competition. The model of perfect competition is based on the following assumptions:

▶ **ASSUMPTION ALERT!** *Assumptions of the model of perfect competition:*

1. *There exist very large numbers of buyers and sellers. No single buyer or seller can influence the price of any good or the actions of other buyers and sellers.*

2. *There exists perfect information, available to all, about the characteristics of all goods and productive inputs. The price of a good contains all the information necessary to judge its utility to any consumer.*
3. *There are no barriers to firms entering any market. Productive inputs are perfectly mobile so that they can migrate to their most productive use.*
4. *All firms within a particular industry are exactly identical. Within a particular industry, there is no reason for consumers to buy one good rather than another except on the basis of price.* ◀

Two critical assumptions of the model of perfect competition are rarely emphasized in economic textbooks: (1) prices carry all the information needed to assign goods and inputs to their most efficient uses and (2) there is no interaction among firms, that is, there is no real competition. Under the assumptions of perfect competition, the prices and quantities produced of each good are set by the forces of supply and demand in a particular industry (widgets, basketballs, or whatever) and those prices are taken by all firms in that industry. As depicted in Figure 4.3, the supply curve for the industry is upward sloping. This is because a higher price encourages more firms to enter the industry and thus increases the total output of the industry. The is sometimes called the **law of supply**, that is, price and quantity supplied or directly related, or $dQ_s / dP > 0$. The demand curve is downward sloping for two reasons: (1) as the price of good increases, consumers switch to other products and (2) as prices increase, a consumer's real income decreases and the consumer purchases less of the good (if the good is a "normal" good, see the discussion of the income and substitution effects in the next chapter). The **law of demand** states that the price and the quantity supplied of a good are inversely related, or $dQ_d / dP < 0$.

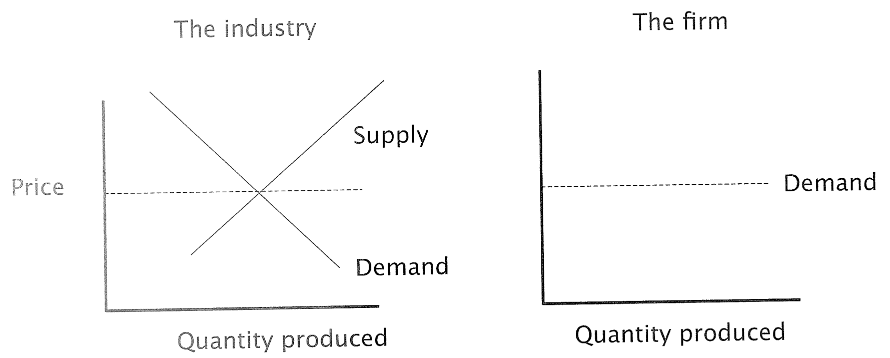


Figure 4.3. **Industries and firms in a competitive market**

In a perfectly competitive market, the firm is a price taker, that is, the price of the good is outside the control of the firm. There is no incentive for a firm to change its price. If it were to raise its price, given the assumptions of the model, its sales would fall to zero because consumers could buy an identical good from another firm at a lower price. There is no reason for a firm to lower its price because it can sell all it produces at the prevailing market price. In economic jargon, the demand curve for a competitive firm is perfectly elastic.

The properties of this model are discussed in countless textbooks and they will not be repeated in detail here. To continue our discussion of Pareto optimality and competition, the main result of this model is that in long-run competitive equilibrium the marginal cost of producing a good is exactly equal to the marginal revenue gained from selling it. This occurs at point e in Figure 4.4. Anywhere to the right of point e, the costs of producing one more unit of the good are higher than the revenue obtained from selling it. Anywhere to the left of point e, the cost of producing one more unit is less than the revenue obtained from selling. So if the firm is producing any output not exactly equal to the equilibrium quantity Q^e, then there is an incentive for the firm to change its output and move to that point.

Firms in long-run competitive equilibrium do not make an **economic profit,** defined as any profit over and above the average rate of profit prevailing in the economy. All firms make the same **accounting profit,** which does not include the **opportunity cost** of producing in the next most profitable industry. For example, if a firm makes an accounting profit of 10 percent and the average rate of profit is 9 percent, then that firm is making an economic profit of 1 percent (accounting profit minus opportunity cost). A firm earning 8 percent would have an economic loss of 1 percent. It could be making more money by moving to another industry.

Another important property of the competitive model is that the firm is producing at the minimum point on the long-run average cost (LRAC) curve.

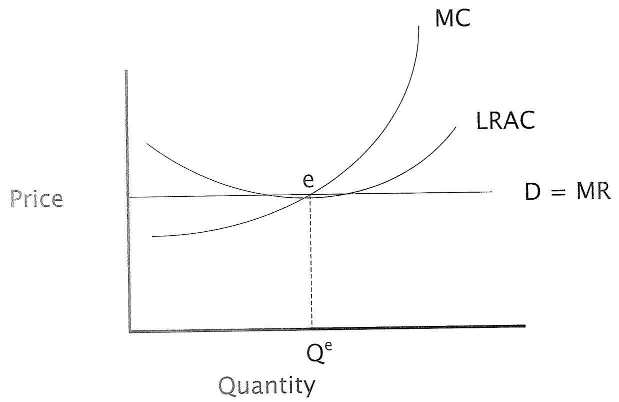


Figure 4.4. Long-run competitive equilibrium

That is, the firm is producing the product at the lowest possible cost, given society's resources and technology. Given the assumptions of the model, every firm in a particular market must be using the most efficient production technique possible or it will be driven out of business by other more efficient firms that can produce the same good at a lower cost and sell it for a lower price. At the minimum point on the long-run average cost curve, that curve is just tangent to the (horizontal) demand curve. If the LRAC curve were above the demand curve, the firm would be operating at a loss because the unit cost of producing the good would be higher than the price received for the good. Firms would leave the industry and the price of the good would rise until the LRAC was just tangent to the demand curve. If the LRAC curve were below the demand curve, the firm would be operating at a profit because the unit cost of producing the good would be lower than the price received for the good. In this case firms would enter the industry, thereby lowering the market price of the good until the demand curve for the firm becomes just tangent to the LRAC curve.

So under the assumptions of perfect competition, market forces ensure that (1) the firm is operating as efficiently as possible, and (2) the price of the good is exactly equal to the marginal cost of producing it.

(4.11) MC=P under the assumptions of perfect competition.

PROOF OF THE PARETO EFFICIENCY OF PERFECT COMPETITION

Recall from Chapter 3 that the basic condition for Pareto optimality in a barter economy is that the (common) marginal rates of substitution between the two goods for the two consumers is equal to the rate of product transformation of the two goods, or

$$(3.2)\qquad MRS_{Y\text{ for }X}=RPT_{Y\text{ for }X}$$

Next, we saw above that

$$(4.3)\qquad P_Y/P_X=MU_Y/MU_X=MRS_{Y\text{ for }X}$$

and

$$(4.10)\qquad RPT_{Y\text{ for }X}=MC_Y/MC_X$$

Combining these equations gives us the proof that Pareto efficiency occurs when:

$$(4.12)\qquad P_Y/P_X=MC_Y/MC_X$$

This is the defining property of a perfectly competitive economy. This result is called the First Fundamental Theorem of Welfare Economics.

The First Fundamental Theorem of Welfare Economics

Given the assumptions of the model of perfect competition, and if all consumers and firms are selfish price takers, a competitive economy must be Pareto efficient.

THE FACTOR MARKET

To complete our discussion of competitive equilibrium we should mention the *factor market*, that is, the demand and supply of factors of production. Using labor as an example, Figure 4.5 illustrates factor demand in a competitive market. The wage rate is set by the forces of supply and demand in the

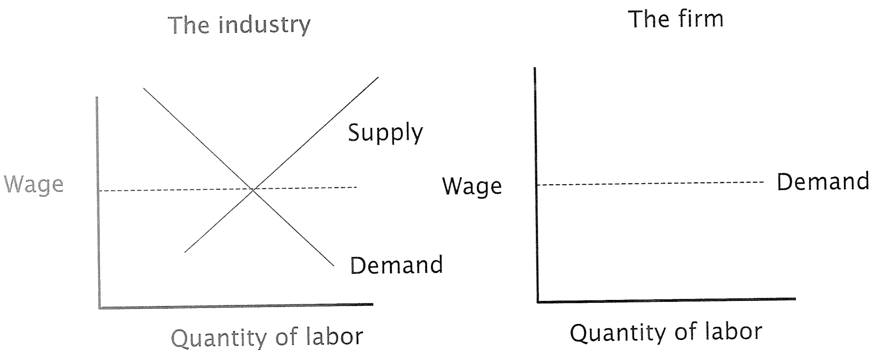


Figure 4.5. Factor demand in a competitive market

labor market for the industry and that wage rate is *given* for the firm. The firm is such a small portion of the industry that it can hire or fire as many workers as it wants to without affecting the wage rate. The demand curve for labor for a firm in a competitive labor market is a horizontal line. In economic jargon, the demand for labor is *perfectly elastic.*

To determine how much of an input a firm will employ, we need two more pieces of information. The demand for a factor of production is a ***derived demand***. It depends not only on the efficiency of the input in producing the good in question, but it also depends on the value of the good being produced. So we need to know the price of the good being produced and the marginal physical product (MPP) of labor (or whatever factor input we are considering). If the marginal physical product of an additional worker is two widgets, and the price of a widget is $5, then the value of the marginal product (VMP) of an additional worker is $10 and that is the equilibrium wage rate. Under conditions of perfect competition in the goods market the price of widgets would be constant at $5. The marginal physical product is declining because of the **law of diminishing returns**. As more and more units of labor are applied to the other fixed inputs, the MPP will eventually decline. So the VMP curve is downward sloping. In equilibrium, we have

$$\text{VMP} = \text{P}_{\text{good}} \text{ X } \text{MPP}_{\text{factor}} \quad (4.13)$$

In equilibrium, the wage is equal to the value of the marginal product. As shown in Figure 4.6, this determines the equilibrium number of workers employed,

here labeled Le. Another "law" of factor demand is that, in a competitive factor market, the ratio of the factor prices should equal the ratio of factor marginal products.

This is illustrated in Figure 4.7 with the two factors capital and labor. In this case, output is maximized at the point where the isoquant is just tangent to the *isocost curve* ($C=P_KK+P_LL$). As we saw in Chapter 2, equation (2.4),

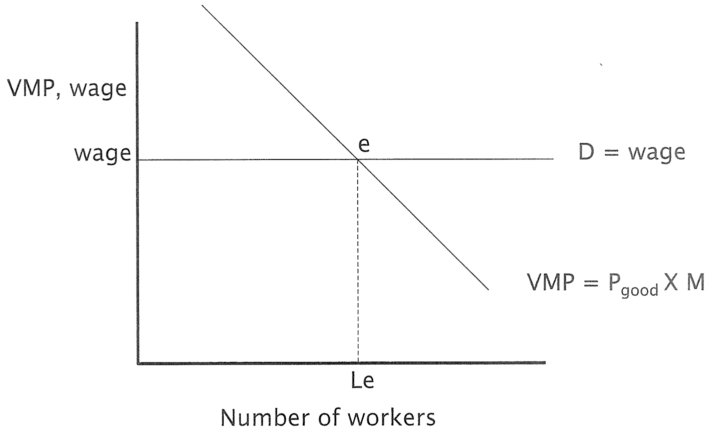


Figure 4.6. The value of the marginal product rule

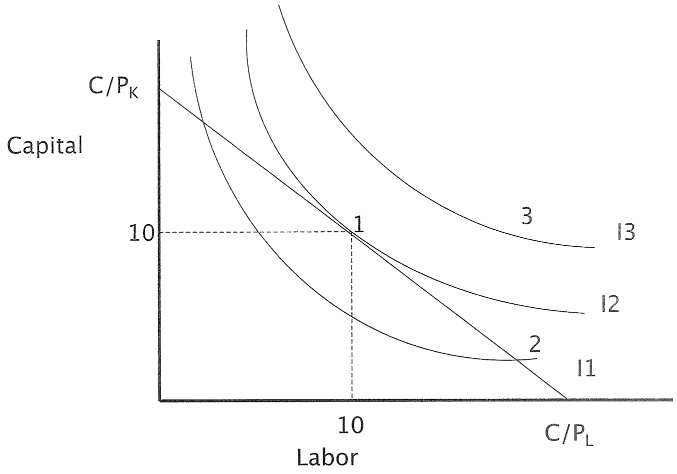


Figure 4.7. Maximizing output subject to a cost constraint

the slope of the isoquant, dK/dL, is equal to the ratio of marginal products MP_L/MP_K. The equation for the isocost curve is $K=C/P_K-(P_L/P_K)L$ and its slope dK/dL is $-P_L/P_K$. So maximizing output given factor cost constraint occurs when

(4.14) $P_L/P_K=MPL/MPK$, or

(4.15) $MPK/P_K=MPL/P_L$

By convention the wage rate P_L is usually denoted by w, and the price of capital, P_K, is usually denoted by r (the rental price of capital).

Prices and the Model of a Barter Economy

According to Walrasian theory, given the same initial conditions, a perfectly competitive economy will achieve the same Pareto efficient outcome as would happen in a face-to-face barter economy. This system places an extraordinary burden on prices *to contain all the necessary information about goods and factors of production. A Pareto efficient outcome also depends critically on the ability of consumers and producers to respond rationally and consistently to these price signals.*

THE CLARK–WICKSTEED PRODUCT EXHAUSTION THEOREM

In Chapter 2 we saw that under competitive conditions in the long run, factors of production are paid according to their marginal physical products. According to the results of Euler's theorem, equation (2.23), this means that the total output produced is exactly used up if it is distributed to the factors producing that output according to the condition: factor reward=marginal product. This is a *steady-state system*. If the economy is producing only corn, and if the factors of production are paid in corn, then at the end of the production period the amount of corn produced will be exactly used up in factor payments.

When we add prices to our model this means that the factor price will equal the value of the marginal product. But this is true only if this economy is characterized by constant returns to scale. This condition holds under the assumptions of perfect competition, as can be seen in Figure 4.4 showing long-run competitive equilibrium. As shown in the figure, a competitive firm will operate at the minimum point on the long-run average cost curve. To the left of this point the economy is characterized by increasing returns to scale (decreasing unit costs), and to the right of this point it is characterized by decreasing returns to scale (increasing costs). So when the economy is in long-run competitive equilibrium it is characterized by constant returns to scale, and there is no "adding-up" problem. If factors are paid according to the value of their marginal products, then the value of total output is exactly equal to the value of the contributions of the factors of production. This is known as the Clark–Wicksteed product exhaustion theorem, named for its formulators, J. B. Clark and Phillip Wicksteed, in the 1890s.

WALRASIAN ECONOMICS IN PRACTICE

It is easy to see why the Walrasian system has held sway for so long. It is nothing less than a complete and logically consistent (if one accepts all the underlying assumptions) representation of Adam Smith's invisible hand. The mathematical form of the system lends itself to easy statistical tests of different questions about the substitutability of goods and inputs, the efficiency of particular markets, and the rate of technological change. But the validity of applying the Walrasian model depends on critical assumptions about human nature, the physical representation of technology, and the characteristics of particular markets. And contrary to the widely held opinion that all models contain assumptions and therefore the realism of assumptions is irrelevant, these assumptions do matter. The question is whether or not the models of *Homo economicus* and perfect competition can be used to make accurate predictions about real-world market behavior. This is examined in Part Two of this book.

Economists are quick to point out that the field is changing rapidly and that much theoretical work being done today does not accept the basic premises of the Walrasian competitive model. This is true. But the basic tools widely used by economists implicitly incorporate the assumptions of that model. The Walrasian model is still the backbone of contemporary economics. *The* *major tools of economic analysis are derived from the Walrasian model and the equilibrium conditions associated with it.* To help understand how the model works in practice, it is useful at this point to consider some specific applications. Here are some examples.

Price and Income Elasticities

Consumer demand for a good in the Walrasian model is a function of the prices of available goods and the consumer's income (M=budget). Consider the demand for good X in the two-good economy:

$$(4.16) \quad Q_X = f(P_X, P_Y, M)$$

Assume there is no **money illusion**, that is, a proportionate change in all prices and income will have no effect on the quantities of X and Y purchased. This is called *homogeneity of degree zero.* In the consumer demand system described above, all the relevant relationships are in terms of ratios. In Figure 4.1 the intercepts of the budget line are M / P_X and M / P_Y, and the common slope of the budget line and isoquant is P_Y / P_X. Multiplying all three terms by some constant does not affect their values.

▶ ASSUMPTION ALERT! *Money and the Barter Economy*

Two assumptions have the effect of ensuring that a competitive money economy will operate in the same fashion as the barter model developed in the first three chapters:

1. *All relevant information about a good or an input can be captured by its price.*
2. *There is no money illusion. A 10 percent increase in the supply of money will result in a 10 percent increase in all prices. Relative prices are insensitive to changes in the amount of money in circulation.* ◀

By Euler's theorem, homogeneity of degree zero implies:

$$(4.17) \quad (\Delta Q_X / \Delta P_X)\, P_X + (\Delta Q_X / \Delta P_Y)\, P_Y + (\Delta Q_X\ / \Delta M)\, M \equiv 0$$

Dividing all the terms on the left-hand side of equation (4.16) by Q_X gives the **own-price elasticity** (E_{XX}), the **cross-price elasticity** (E_{XY}), and the **income elasticity** (E_{XM}). It also reveals the relationship (given the negativity of own-price elasticity):

$$(4.18) \quad E_{XX} = E_{XY} + E_{XM}$$

The value of the own-price elasticity of a good is equal to the sum of the income elasticity and cross-price elasticities. Restrictions such as equation (4.18) are routinely exploited by economists in econometric work. The assumptions built into these relationships are rarely discussed.

The Elasticity of Substitution

As Chapter 2 showed, the elasticity of substitution measures the ability of an economy to substitute one input for another as the relative values of the marginal products of the inputs change.

$$(4.19) \quad \sigma = \Delta(K / L)/(K / L) \div [\Delta(MRTS_{L\ \text{for}\ K})\ / MRTS_{L\ \text{for}\ K}]$$

Because in competitive equilibrium $MRTS_{L\ \text{for}\ K} = MP_L / MP_K$, the elasticity of substitution is usually written

$$(4.20) \quad \sigma = \%\ \Delta(K / L) \div \%\ \Delta[(P_L / P_K)]$$

Equating the ratios of marginal products to the ratio of factor prices is justified only if the factor market is in competitive equilibrium. This condition requires perfect competition in the goods market and factor market, constant returns to scale, smooth and continuous isoquants, and the output maximization condition. Measures of the elasticity of substitution are widely used in public policy debates. For example, the degree of substitutability between energy and capital has been the topic of many econometric studies. The effects of the built-in assumptions are seldom considered.

Total Factor Productivity

Total factor productivity (TFP) is an attempt to measure the effect of pure technological change on output growth. It is written as:

$$(4.21) \quad A = \dot{Q} - a\dot{K} - (1 - a)\dot{L}$$

A dot over an element in equation (4.21) indicates a rate of growth. The weights a and (1 − a) are the product shares of capital and labor. In econometric studies estimating TFP, each input is weighted using its cost share rather than its share of the output of the product (in the case of labor this is wL / C). This approach is valid only under conditions of perfect competition in both the goods and factor markets. Perfect competition implies constant returns to scale, constrained optimization, and the assumptions discussed above (see the discussion of the **dual** below). How these assumptions affect the validity of using cost shares as weights is rarely discussed in TFP studies.

APPENDIX

Constrained Optimization and the Lagrangian Multiplier

Constrained optimization is the maximization (or minimization) of some objective function subject to constraints on the independent variable. Consider the problem of maximizing utility subject to a budget constraint. A specific utility function and budget constraint might be:

(4.22) $U = X_1X_2 + 4X_2$ subject to the budget constraint $2X_1 + X_2 = 30$

The Lagrangian version of this would be:

(4.23) $Z = X_1X_2 + 4X_2 + \lambda(30 - 2X_1 - X_2)$

The idea is to make the constraint equal to zero so that we can evaluate the utility function without worrying about constraints. In that case U will be equal to Z. This is where λ comes in. We treat λ as an additional variable, that is, instead of the function $U = f(X_1, X_2)$, we have $Z = f(\lambda, X_1, X_2)$. Then the first-order conditions are:

(4.24) $\partial Z / \partial X_1 = X_2 + 2\lambda = 0$

(4.25) $\partial Z / \partial X_2 = X_1 + \lambda = 0$

(4.26) $\partial Z / \partial \lambda = 30 - 2X_1 - X_2 = 0$

By treating λ as an extra variable and solving the system of equations including it, we can treat the values of Z as if they were free (unconstrained) variables.

Kuhn–Tucker Conditions

The mathematics we used above to describe the Pareto conditions for efficiency in exchange are designed for problems of constrained optimization. For example, we began by deriving the conditions for maximizing utility subject to a given endowment of goods X and Y. The Kuhn–Tucker conditions are a way of writing down the constrained optimization problem when there are non-negativity constraints. For example, in the graphical representation of the consumer maximization problem, it is necessary for the consumer to consume positive amounts of both goods or else we might have a situation illustrated in Figure 4.8.

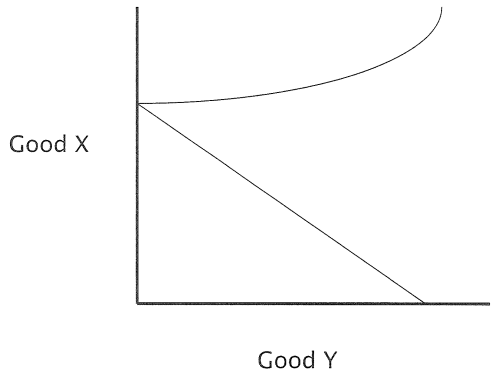


Figure 4.8. A corner solution

In this case the consumer is spending his or her entire budget on good X and it is impossible for the indifference curve to be tangent to the budget line. The consumer's constrained maximization problem becomes:

(4.27) $Z = U(X, Y) - \lambda(P_XX + P_YY - B)$

$B \le P_XX + P_YY$

$X, Y \ge 0$

The first-order conditions become:

(4.28) $\partial Z / \partial X = \partial U / \partial X - \lambda P_X \le 0$ if <, then $X = 0$

(4.29) $\partial Z/\partial Y=\partial U/\partial Y-\lambda P_Y \leq 0$ if <, then $Y=0$

(4.30) $\partial Z/\partial \lambda=B-P_X X-P_Y Y \geq 0$ if >, then $\lambda=0$

If $X, Y>0$, then $MU_X=\lambda P_X$ and $MU_Y=\lambda PX$, and $\lambda=MU_X/P_X=MU_Y/P_Y$.

If the total budget is not spent, and there is some income left over after the consumer has all the X and Y desired, then the marginal utility of income λ must be zero.

The Dual and the Function Coefficient

There is an important relationship between cost functions and production functions called the dual. Figure 4.7 shows the conditions for output maximization given a cost constraint. We can arrive at the same result if we start with a production function (an isoquant) and see how we can produce a given amount of output with minimum cost (the lowest isocost curve). We could produce the given output using the quantities of labor and capital indicated by points 1 or 2 on isocost curve I3 (see Figure 4.9), but we can produce it cheaper by using the combination of labor and capital indicated by point e on isocost curve I2. With isocost I1 we do not have enough money to purchase the inputs necessary to produce the quantity of output indicated by the isoquant.

So whether we start with the isoquant and minimize costs, or start with the isocost curve and maximize output, we arrive at the same optimal amounts of inputs K and L. In economic theory this is called the dual. As we saw above, this means that, with competitive equilibrium in production, the slope of the isoquant is tangent to the isocost curve, so that the ratio of factor prices is equal to the ratio of marginal products. This is a particularly important result because, with the equilibrium assumption, it is possible to start with a production function and go directly to a cost function.

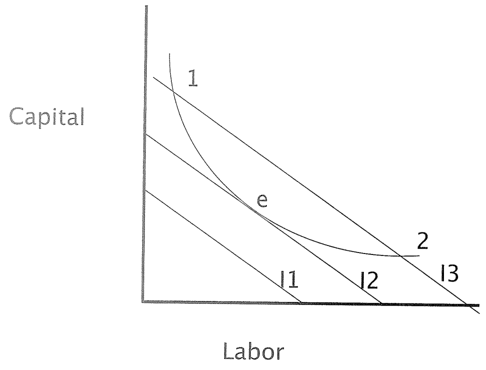


Figure 4.9. **Minimizing costs subject to an output constraint**

The **function coefficient** is the proportional change in output resulting in an equal proportional change in all inputs. Given two inputs K and L and output Q, we have:

(4.31) $\varepsilon=(\Delta Q/Q)/(\Delta K/K)+(\Delta Q/Q)/(\Delta L/L)=(\Delta Q/\Delta K)/(K/Q)$
$+(\Delta Q/\Delta L)/(L/Q)$, or

(4.32) $\varepsilon=MP_K(K/Q)+MP_L(L/Q)$

Now multiply the first term on the left-hand side by $1=P_K/P_K$ and the second term by $1=P_L/P_L$. This gives us:

(4.33) $\varepsilon=(MPK/P_K)\ (KP_K/Q)+(MPL/P_L)\ (LP_L/Q)$

We saw above in equation (4.15) that equilibrium in the factor market requires that $(MPK/P_K)=(MPL/P_L)$, so we can rewrite equation 4.33 as

(4.34) $\varepsilon=(MPK/P_K)[(KP_K+LP_L)/Q]$

We saw above that the cost constraint $C=(KP_K+LP_L)$ and we know that C/Q is the average cost of producing a unit of Q, so we can write 4.34 as

(4.35) $\varepsilon=(MPK/P_K)(AC)$

The last thing we need to do is to show that (P_K/MPK) is equal to long-run marginal cost LMC. Suppose we wanted to produce one more unit of Q and that the marginal product of capital is five. How many units of capital would be needed? The answer is 1/5. How much would that cost? The answer is $(P_K)(1/MPK)$. So the function coefficient can be written as

(4.36) $\varepsilon=LAC/LMC$

We began with a production function and ended up with an expression made up only of costs. We did this by making the critical assumption that the factor market was in competitive equilibrium so that we could convert marginal products to marginal costs. This is critically important in econometric studies because there is no data on marginal products of inputs, but there is data on wages, rent, interest, and other input costs. The graphical result above showing the duality between cost and production can be proved mathematically using *Shephard's Lemma*, an application of a mathematical relationship called the *envelope theorem*.

The Theory of the Second Best

If several optimality conditions in an economic model are not satisfied, would we move toward Pareto efficiency by correcting only one of them? The surprising answer is no. It is not only possible but very likely that correcting one of the imperfections will move us farther away from the competitive ideal. This result is called the theory of the second best, as first demonstrated by R. G. Lipsey and Kevin Lancaster in the 1950s. The very practical policy implication of the theory is that economists need to carefully examine the specific characteristics of a particular market before making theory-based "first-best" recommendations about how to improve economic efficiency. Richard Howarth examined the effects on the economy of environmental taxes and found that first-best rules underestimate the optimal level of emissions taxes under a variety of policy scenarios (see Howarth 2005).

GLOSSARY

Accounting profit—The actual profit a firm makes, that is, the total revenue from selling the goods produced minus the total costs of producing them. Accounting profit does not include opportunity costs.

Cross-price elasticity—The effect of a change in the price of one good on the quantity demand of another. The effect is negative for goods that are complements (i.e., used together, such as gin and tonic) and positive for goods that are substitutes (such as apples and oranges).

Derived demand—The market for a factor of production depends not only on its productivity but also on the demand for the product it is producing.

Dual—Maximizing output subject to a cost constraint or minimizing costs subject to an output constraint gives the same optimal combinations of inputs.

Economic profit—The profit a firm makes minus the average profit prevailing in the economy. In long-run competitive equilibrium, profits and losses are eliminated by the forces of competition so that economic profit is zero for all firms. That means all firms are making the same accounting profit.

First-order conditions—When a consumer is maximizing utility subject to a budget constraint, the ratio of marginal utilities will equal the ratio of the prices of the goods. When a producer is maximizing output subject to a cost constraint, the ratio of the marginal products of the factors of production will equal the ratio of their prices.

Function coefficient—The proportional change in output resulting in an equal proportional change in all inputs.

Income elasticity—The effect of a change in income on the demand for a good. The effect is positive for normal goods (your income goes up so you buy more of it) and negative for inferior goods (your income goes up and you buy less of it).

Kuhn-Tucker conditions—A way of stating the first-order conditions for a constrained optimization problem when there are non-negativity constraints on the variables to be maximized.

Law **of demand**—The price of a good and the quantity demanded of it are inversely related, or $dQ_d / dP < 0$.

Law **of diminishing returns**—As more and more units of a productive factor are added, the amounts of all other factors held constant, the marginal physical product of that factor will eventually decline.

Law **of supply**—The price of a good and the quantity of it supplied are directly related, or $dQ_s / dP > 0$.

Marginal utility of money—When the consumer is maximizing utility subject to a budget constraint, the consumer gets the same utility from a dollar spent on each good. So in equilibrium, $\lambda = MUx / Px = MU_Y / P_Y$ can be interpreted as the marginal utility of money.

Money illusion—The failure to distinguish between nominal (actual) and real (adjusted for inflation) prices.

Opportunity cost—The cost foregone by not choosing the next best alternative. For a firm, it is the profit foregone by not producing in the next most profitable industry.

Own-price elasticity—The effect of an increase in the price of a good on the quantity of it demanded.

Production possibilities frontier—Shows the maximum amount of one good that can be produced, given the amount produced of the other good. On the frontier, resources and technology are used to their maximum efficiency.

Rate of product transformation—The rate at which the resources used in the production of one good can be shifted to the production of another good while maintaining the greatest output efficiency possible. It is equal to the slope of the production possibilities frontier.

Second-order condition—Necessary to ensure that we are obtaining a maximum value and not a minimum value.

REFERENCE

Howarth, R. 2005. The present value criterion and environmental taxation: The suboptimality of first-best decision rules. *Land Economics* 81, 321–336.

5

MARKET FAILURE AND THE SECOND FUNDAMENTAL THEOREM OF WELFARE ECONOMICS

What is it we mean by "market failure"? Typically, at least in allocation theory, we mean the failure of a more or less idealized system of price-market institutions to sustain "desirable" activities or stop "undesirable" activities. The desirability of an activity, in turn, is evaluated relative to the solution values of some explicit or implied maximum welfare problem.

—*Francis Bator, "The Anatomy of Market Failure,"* Quarterly Journal of Economics *72(3) (1958), 351*

The model of perfect competition presented in the last chapter is the heart and soul of neoclassical welfare economics. It is a mathematical representation of an economic system that exactly duplicates the operation of a voluntary, frictionless, barter economy. As we saw in Chapter 1, the starting point for the model is the individual consumer whose behavior is described by the rational actor model. The consumer chooses from among the array of market goods based solely on the prices of those goods, and producers supply these goods using the most efficient combinations of inputs as indicated by their prices (see Figure 5.1). Two critical assumptions drive the demand for market goods in this model. The first is the assumption of rational choice on the part of consumers and producers, and the second is faith in the ability of prices to correctly capture all the relevant information about market goods. If the price signals are "wrong," then so too will be the collection of market goods chosen by the rational consumer.